Efficient Sorting and Searching in Rendering Algorithms

Eurographics 2006 Tutorial T4

Organizers and Presenters

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Abstract

In the proposed tutorial we would like to highlight the connection between rendering algorithms and sorting and searching as classical problems studied in computer science. We will provide both theoretical and empirical evidence that for many rendering techniques most time is spent by sorting and searching. In particular we will discuss problems and solutions for visibility computation, density estimation, and importance sampling. For each problem we mention its specific issues such as dimensionality of the search domain or online versus offline searching. We will present the underlying data structures and their enhancements in the context of specific rendering algorithms such as ray shooting, photon mapping, and hidden surface removal.

Organizers bibliographies

Vlastimil Havran is an assistant professor at the Czech Technical University in Prague since February 2006. He defended his Ph.D. dissertation on ray shooting algorithms in 2001 at the Czech Technical University in Prague. Later he joined the computer graphics group at Max-Planck-Institute for Informatics in Saarbruecken. He became a research associate at the same institute in 2003. He has contributed to the topic of sorting and searching by his dissertation on ray shooting algorithms which started the area of interactive ray tracing. In addition to sorting and searching he worked on various other topics in rendering.

Jiří Bittner holds a Ph.D. in Computer Science from the Czech Technical University in Prague. His main research interests include visibility preprocessing, occlusion culling, real-time rendering, and computational geometry. He has also been active in development of two commercial products dealing with real-time rendering of large scenes. He is currently affiliated with the Vienna University of Technology and the Czech Technical University in Prague.

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Tutorial Contents

Tutorial Web Page

The updated version of this tutorial presented at Eurographics 2006 can be found under the following URL:

`http://www.cgg.cvut.cz/~havran/eg2006tut/`

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Content

Part One

- Introduction to Rendering
- Sorting and Searching
- Hierarchical Data Structures
- Ray Shooting
- Questions and Answers (5 minutes)

Content

Part Two

- Hidden Surface Removal
- Visibility Culling
- Photon Maps and Irradiance Caching
- Ray Maps
- Other Algorithms
- Questions and Answers (10 minutes)

Tutorial Goals

- Recall that we often use sorting and searching in rendering
- Highlight connections between different problems in rendering
- Briefly show efficient solutions
- Show unusual solutions resulting from twisting searching queries and domains

What is Not Covered Here

- Collision detection algorithms (EG'05 Tutorial)
- Image based rendering
- Radiosity
- Non-photo realistic rendering
- Clustering techniques
- Graph theory and other related problems

Updated tutorial slides available at
http://www.cgg.cvut.cz/~havran/eg2006tut/

Introduction to Rendering

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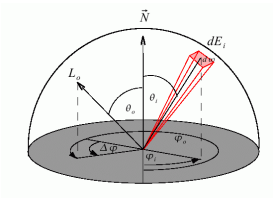
Introduction to Rendering

- Rendering equation
- Rendering algorithms
- Sorting and searching in rendering

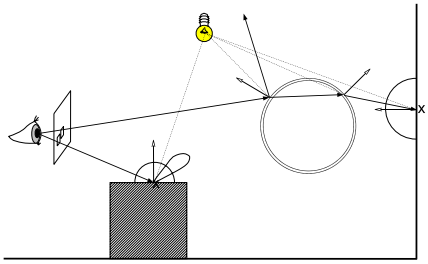
Rendering Equation

$$L(\vec{\omega_o}, x) = L^e(\vec{\omega_o}, x) + \int_\Omega L_i(x, \vec{\omega_i}) \cdot f_r(x, \vec{\omega_i}, \vec{\omega_o}) \cdot (\vec{n} \cdot \vec{\omega_i}) d\omega_i,$$

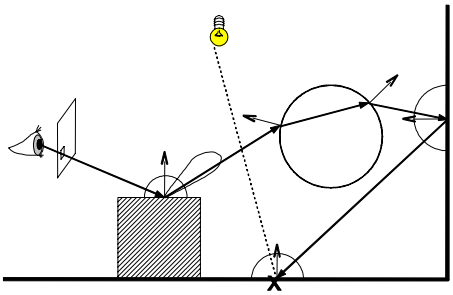
- Convolving incoming light with surface reflectance properties



Ray Tracing

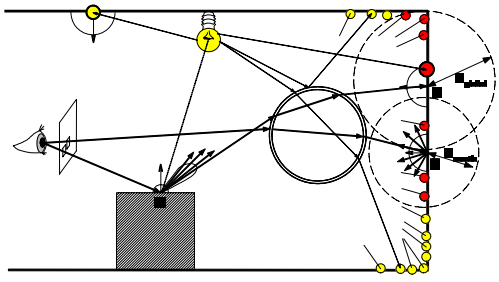


Path Tracing



Photon Mapping

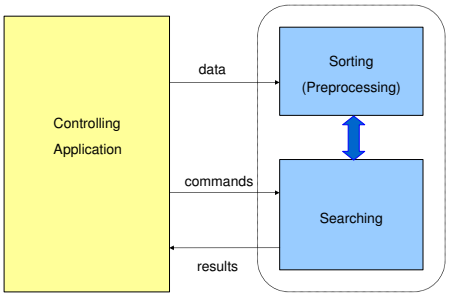
Phase I: photon shooting Phase II: gathering



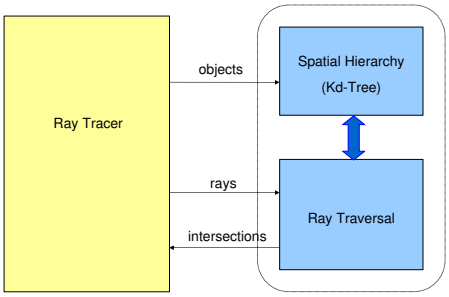
Tutorial Motivation

- Sorting and searching takes usually more than 90% of the rendering time!
- Efficiency of sorting and searching is crucial for the performance
- Examples
  - Ray Tracing
  - Photon Density Estimation

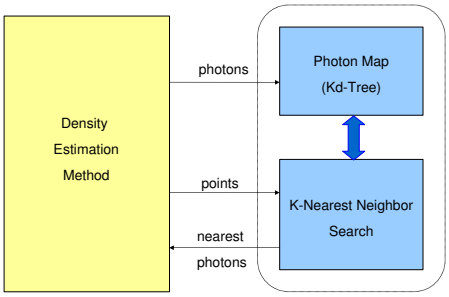
Sorting and Searching General Concept



Example 1 Ray Tracing



Example 2 Photon Density Estimation



Introduction to

Sorting and Searching

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Searching

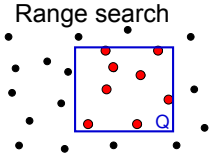
- Search problem

$$Q \times S \rightarrow A$$

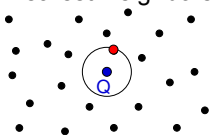
Q query domain
S search space
A domain of answers

Geometric Search Problems

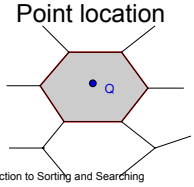
Range search



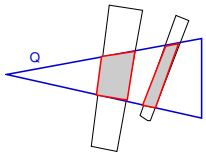
Nearest Neighbors



Point location



Intersection detection



Search Problems in Rendering

Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Searching Algorithms

- Exact vs. Approximate
  - Approximate: finds solution close to exact one
- Online vs. offline
  - Offline: applied for entire sequence of Q
- Static vs. dynamic
  - Dynamic: S may change

Sorting

- Organizing data
- Improve searching performance
  - Naïve search: O(n) time
  - With sorting: O(log n)!
  - In special cases even O(1)
- We deal with multidimensional data!
  - Define relations among elements of S (objects, points, rays, normals, …)

Basic Sorting Algorithms

Algorithm	Method	Best	Average	Worst
Heapsort	Selection	O(n log n)	O(n log n)	O(n log n)
Selection sort	Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Quicksort	Partitioning	O(n log n)	O(n log n)	$O(n^2)$
Bucket sort	Distribution	O(n)	O(n)	$O(n^2)$
Merge sort	Merging	O(n log n)	O(n log n)	O(n log n)
Bubble sort	Exchanging	O(n)	$O(n^2)$	$O(n^2)$
Insertion sort	Insertion	O(n)	$O(n^2)$	$O(n^2)$

Space complexity: O(n)

Sorting in Rendering

- Sort by partitioning (Quicksort like)
  - Top-down construction of spatial hierarchies
- Sort by selection (Heapsort like)
  - Bottom-up construction of spatial hierarchies
  - k-NN search
- Sort by distribution (Bucket sort like)
  - Rasterization
- Sort by exchanging (Bubble sort like)
  - Incremental priority orders

Hierarchical Data Structures

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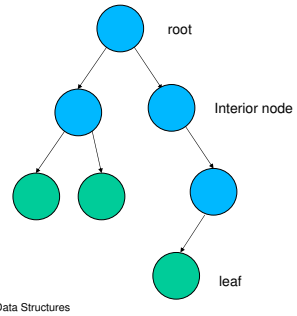
Hierarchical Data Structures (HDS)

- Connection to sorting
- Classification
- Bounding volume hierarchies
- Spatial subdivisions
- Hybrid data structures
- Searching using HDS
- Special techniques on hierarchies

Hierarchical Data Structures

= tree or even a graph



Connection to Sorting

Hierarchical Data Structures =
implementation of (spatial) sorting

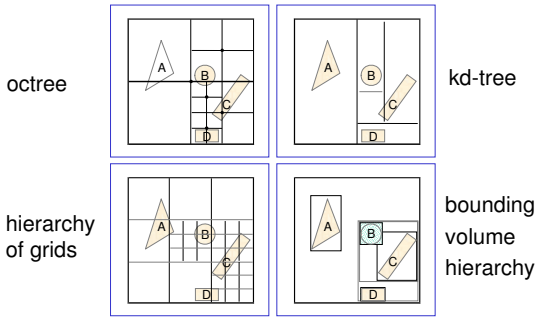
Why ?

- Time complexity is O(N log N)
- For 1D hierarchy over points the construction of HDS is clearly equivalent to quicksort

Recall Quicksort

- Pick up a pivot Q
- Organize the data into two subarrays: the left part smaller than pivot Q, the right part larger or equal than pivot Q
- Recurse in both subarrays

Examples of HDS in 2D



HDS Classification

- Data domain organization
- Dimensionality
- Data layout

HDS Classification

1) Data domain organization of HDS

- Spatial subdivisions – primarily organizing space (non-overlapping)
- Object hierarchies – primarily organizing objects (overlapping)
- Hybrid data structures
- Transformations and mappings

HDS Classification

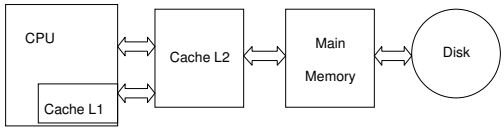
2) Dimensionality of HDS

- Necessary to represent data entities: 1D, 2D, 3D, 4D, or 5D
- Data entities: points, lines, oriented half-lines, disks, oriented hemispheres, etc.
- Possibility to extend many problems to time domain (so plus one dimension)

HDS Classification

3) HDS data layout

- Internal data structures
- External data structures (out of core)
- Cache-aware data structures
- Cache oblivious data structures

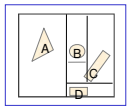


Types of Nodes in HDS

- An interior node represents a *"pivot"* – according to it the data entities are sorted
- Typical content is a subdivision plane or a set of planes plus references to child nodes
- The way of interior node representation with respect to the task is crucial for searching performance

Spatial Subdivisions

- **Non-overlapping *regions of child nodes***
- Space is organized by some (cutting) entities, typically by planes, constructed top-down
- Fully covering an original spatial region, point location always possible in some (empty or non-empty) leaf
- They are often called space partitionings



kd-tree

Spatial Subdivision Examples

- Kd-trees – axis aligned planes
- BSP-trees – arbitrary planes
- Octrees – three axis aligned planes in a node
- Uniform grids (uniform subdivision)
- Recursive grids

Object Hierarchies

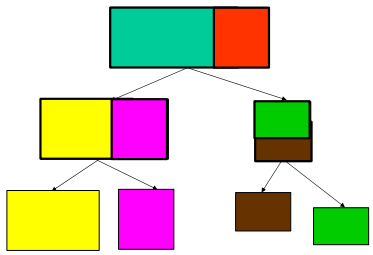
- Possibly **overlapping** extents of child nodes
- Many different names - often called bounding volume hierarchies
- Possibly some spatial regions are not covered (point location impossible)
- Constructed top-down or bottom-up
- The shape represented by interior nodes typically a box, but other shapes as spheres also possible

Names used for Object Hierarchies

- Bounding Volume Hierarchies (BVHs)
- R-trees and their many variants
- Box-trees
- Several others (special sort of bounding volumes... sphere trees etc.)

Bounding Volume Hierarchies

Constructed Top-Down



Hybrid Data Structures

- Combining between various interior nodes
- Possibly combining between spatial subdivisions and object hierarchies
- Sharing pros and cons of both types
- They can be tuned to compromise of some properties, for example efficiency and memory requirements

Other HDS

- *Content of the node* – a single splitting plane, more splitting planes, a box, additional information.
- Arity of a node (branching factor)
- A way of constructing a tree (height, weight balancing) + postprocessing
- Data only in leaves or also in interior nodes
- Augmenting data

Example of Other HDS

- Cell trees (polyhedral shapes for splitting)
- SKD-trees (two splitting planes at once)
- hB-trees (holey brick B-trees)
- LSD-tree (height balanced kd-tree)
- P-trees (polytope trees)
- BBD-trees (bounding box decomposition trees)
- And many others

(See the surveys listed in tutorial notes, in particular [Samet06])

Transformation Approach

- *Input:* A spatial object in 2D or 3D domain, for example a box
- *Output:* A point in 4D or 6D domain
- More complicated mapping is possible, for example a sphere in 3D maps to a 4D point
- The transform often changes the searching algorithm used completely

HDS Construction Algorithm

Initial Phase: create a node with all elements and put it to the auxiliary structure AS (stack or priority queue).

Top-Down, Divide and Conquer:

(1) Take a node from an auxiliary structure AS. If AS is empty, then we are finished.

(2) Take a set of elements in the node and decide if to subdivide or not. If not, create leaf, go to (1).

(3) Decide how to split the set into two (N) subsets and create new nodes.

(4) Put the new nodes to AS. Recurse.

Search Algorithms using HDS

- Start from the root node
- Typically down traversal phase (location phase) + some other phase
- During visiting an interior node use either a stack (LIFO) or priority queue to record the nodes to be visited in future
- Compute incidence (such as ray-object intersection) when visiting a leaf

Note: auxiliary structure implements another sorting phase during searching

Search Algorithms using HDS

- *Range queries* – given a range X, find all the incidences of X with data
- *Nearest neighbour* – find a nearest neighbor
- *k-nearest neighbour*
- *Intersection search* – given a point Q, find all the objects that contain Q
- *Ranking* – given a query object Q, report on all the objects in order of distance from Q
- *Reverse nearest neighbours* – given a point *Q,* find all the points to which Q is nearest neighbour

Search Performance Model

- Result = the cost of computation ... C
- Performance is inverse proportional to the quality of the data structures for given problem
- The two uses of performance model
  - *a posteriori:* documenting and testing performance
  - *a priori:* constructing data structures with higher expected performance

Search Performance Model

Typical cost model:

$$C = C_T + C_L + C_R$$

$$C = C_{TS} * N_{TS} + C_{LO} * N_{LO} + C_{Access} * N_{Access}$$

- C_T … cost of traversing the nodes of HDS
- C_L … cost of incidence operation in leaves
- C_R … Cost of accessing the data from internal or external memory

HDS Dynamization

- Given changes, only update data structures to reflect these changes
- It is assumed that the performance of searching remains acceptable after update
- Dynamization can require additional bookkeeping data to monitor the cost/quality of a HDS node and the subtree associated with the node
- Techniques known for 1D trees (rotation, balancing) are often not applicable
- It is usually required to update larger amount of data at once (*bulk updating*)

Ray Shooting

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Ray Shooting

Find nearest intersection along a ray

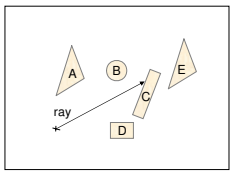
Problem	Q	S	A
Ray shooting	**ray**	**{objects}**	**point**
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Ray Shooting

- Ray shooting versus ray tracing
- Connection to sorting and searching
- Ray shooting with kd-trees
- Performance model/studies
- Octrees, uniform grids, recursive grids
- Bounding volume hierarchies
- Offline ray shooting

Ray Shooting Algorithm (RSA)

Task: Given a ray, find out the first object intersected.



Input: a *scene* and a *ray*

Output: the *object C*

Ray Tracing *versus* Ray Shooting

- *Ray shooting* – only a single ray
- *Ray tracing* in computer graphics can be:
  - Ray shooting (only a single ray)
  - Ray casting – only primary rays from camera
  - Recursive ray tracing
  - Distribution ray tracing and others

Some Complexity Issues

Computational Geometry
- aimed at worst-case complexity
- restriction to certain class of object shape (polygons, triangles)
- unacceptable memory requirements

  O(log N) query time induces $\Omega(N^4)$ space

Computer Graphics
- aimed at average-case complexity
- practical feasibility and robustness
- implementation issues important for performance

Some Complexity Results

Lower bound for worst-case complexity: 1997/98
Laszlo Szirmay-Kalos + Gabor Marton – lower bound for space complexity is $\Omega(N^4)$ for O(log N) search

Applicability of Computational Geometry techniques in CG for ray tracing
- CGE techniques are not general
- limited to small number of primitives
- no implementations available

Computer Graphics Techniques Overview

Techniques developed: aimed at practical applications, no complexity guarantees, many "tricks", the analysis difficult or infeasible

Basic techniques: bounding volumes, spatial subdivision, ray classification

Augmented techniques: macro regions, pyramid clipping, proximity clouds, directed safe zones

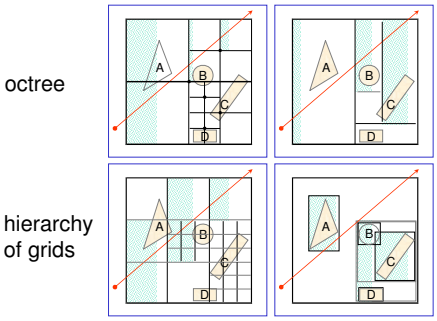
Special tricks: ray boxing, mailbox, handling CSG primitives, other types of coherence

RSA Techniques Classification

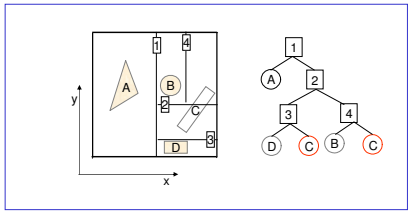
A) Subdivision techniques (top down)
- -- binary space partitioning (kd-trees)
- -- octrees
- -- uniform and hierarchical grids
- -- bounding volume hierarchy

B) Clustering (bottom up)
- -- bounding volume hierarchy

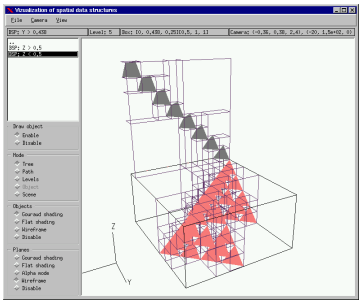
Some RSA Techniques



Kd-tree Construction



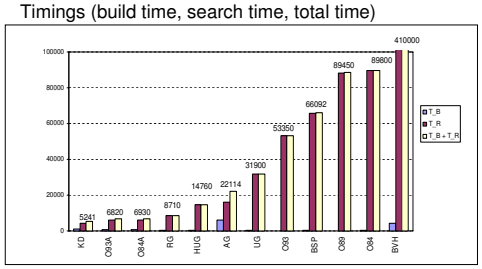
Visualisation of Kd-tree



RSA Techniques Comparison

30 scenes *times* 12 RSAs *times* 4 ray distribution methods
= 1440 measurements, year 2000-2001

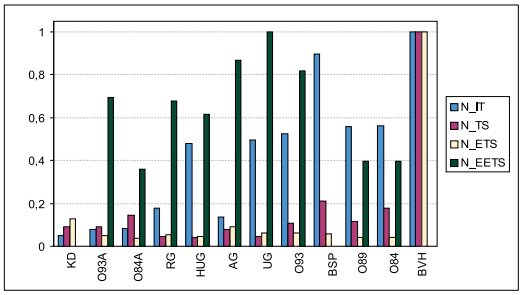
Timings (build time, search time, total time)



Note: In test BVH constructed in bottom-up fashion !

RSA Techniques Comparison

Number of operations (ray-object intersections, traversal steps)



Note: values normalized to the worst value.

RSA based on Kd-tree

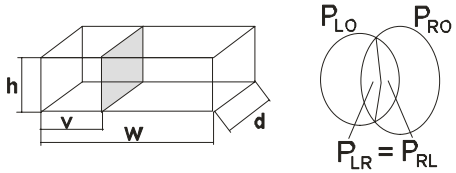
Quite an efficient solution used in practice

Construction (in O(N log N) time)
- based on cost function and geometric probability
- automatic termination criteria algorithm
- various efficiency improvements:
  - construction of kd-tree with empty spatial regions
  - reducing objects' axis-aligned bounding boxes
  - preferred ray sets

Ray traversal
- in practice achieves expected O(log N) time
- robust recursive ray traversal algorithm

Geometric Probability of Ray Intersecting a Subdivided Box

$probability_{LEFT} = P_{LO} + P_{LR} + P_{RL}$
$probability_{RIGHT} = P_{RO} + P_{LR} + P_{RL}$

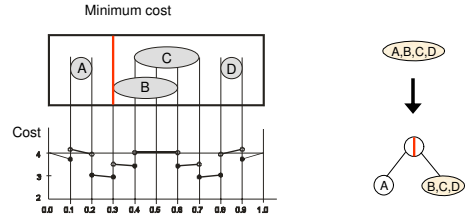


Probability computed from surface area of the box

Condition: uniform ray distribution

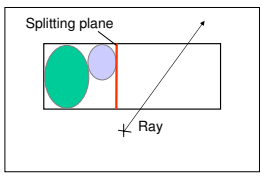
Kd-tree Construction Based on Cost Function with Greedy Heuristics

$Cost = probability_{LEFT} * N_{LEFT} + probability_{RIGHT} * N_{RIGHT}$

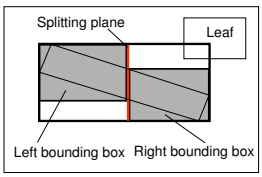


Kd-tree Efficiency Improvements

Cutting off empty space



Reducing objects' axis-aligned bounding boxes

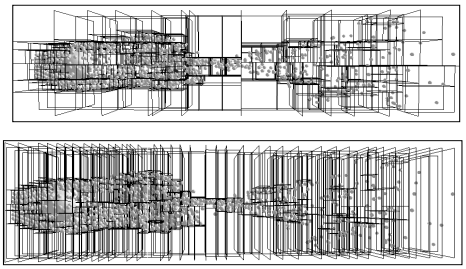


Termination Criteria for Construction

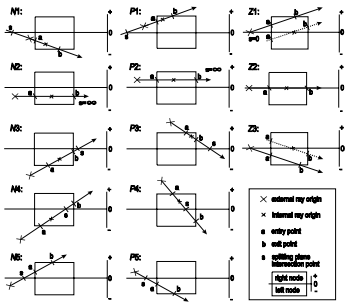
- *Local: using a stack*
  - *Simple local*: maximum depth + number of objects
  - *More complicated local*: a maximum number of cost improvement failures + maximum estimated depth + number of objects
- *Global: using a priority queue*
  - maximum memory used
  - maximum memory used + maximum leaf cost

Kd-tree Construction for Preferred Ray Sets

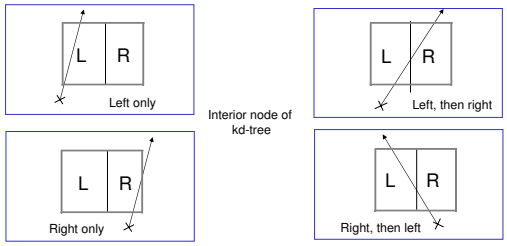
Idea: different than uniform distribution of rays, gain 2-3



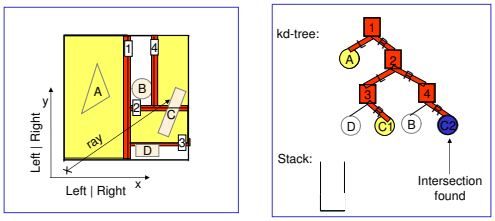
Recursive Ray Traversal Algorithm for Kd-tree



Recursive Ray Traversal Basic Cases Classification

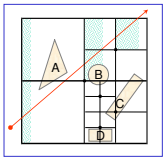


Recursive Ray Traversal Algorithm



Ray Shooting with Octrees

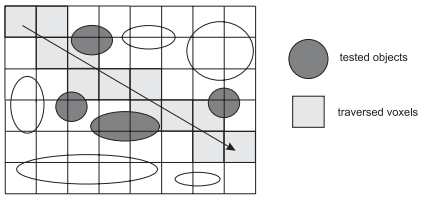
- Interior node arity is eight
- Up to four child nodes can be traversed in an interior node
- Traversal algorithm necessarily more complicated than for kd-tree
- About 26 papers about ray tracing with octrees were published
- Octrees are less adaptive to the scene object distributions than kd-trees
- Geometric probability can be used in the same way as for kd-trees (Octree-R)
- According to the experiments, octrees are less efficient than kd-trees



Note: octrees can be simulated by kd-trees

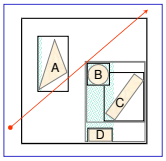
Ray Shooting with Uniform Grids

- Arity of a node proportional to the number of objects
- Traversal method based on 3D differential analyzer (3DDA)
- For skewed distributions of objects in the scene it is inefficient
- For highly and moderately uniform distributions of objects it is slightly more efficient than kd-trees



Ray Shooting with Bounding Volume Hierarchy (BVH)

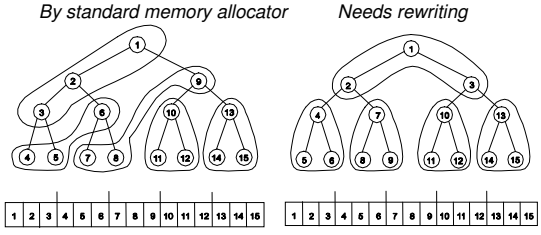
- Each interior node is fully described by a bounding box
- The number of child nodes is usually two for top-down construction (more for bottom-up construction)
- The nodes can overlap
- Each object is referenced only once
- The storage of a node is high … the memory consumption is higher than for kd-trees
- Traversal algorithm similar to kd-trees
- Kd-trees can be emulated by BVHs.



Data Layout in Memory

Inorder, preorder (depth-first-search), heap (bread-first-search), van Emde Boas

By standard memory allocator *Needs rewriting*



Depth-first-search (DFS) Van Emde Boas

Performance Model of Ray Shooting

Total cost for RSA =
cost for ray-object intersection tests +
cost for ray traversal of kd-tree +
cost for data move from memory to CPU

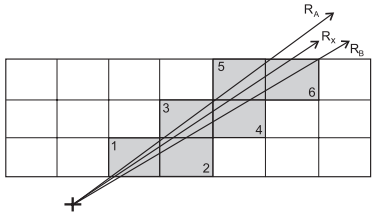
- Faster ray-object intersection tests
- Decreasing number of ray-object intersection tests
- Faster traversal step
- Decreasing number of traversal steps
- Reducing CPU-memory traffic

Offline Ray Shooting

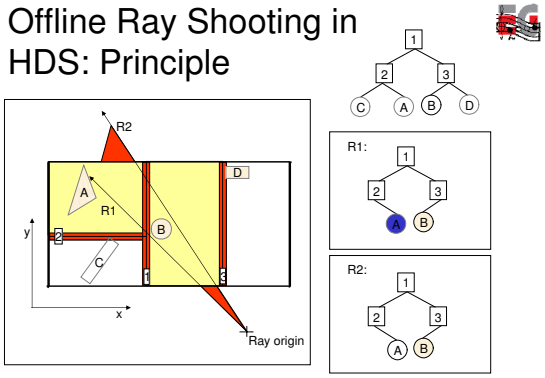
- Shooting several rays at once
- Rays are formed by camera, by viewing frustum or by point light sources
- Rays are **coherent** = similar in direction and origin
- Problem can be formulated as offline setting of searching
- We can amortize the cost of traversal operations though the data structure … the number of traversal steps is decreased typically by 60-70%
- Solving by LCTS – longest common traversal sequence

Offline Ray Shooting: Coherence

- If boundary rays traverse the same sequence S of leaves, then all rays in between also traverse the same sequence.
- Proof by convexity (convex leaves, convex shaft)



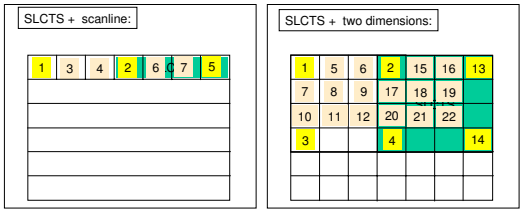
Offline Ray Shooting in HDS: Principle



Ray Shooting 31

Sampling in Image Space

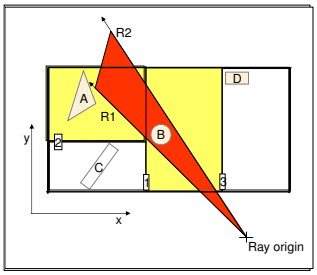
Hidden surface removal based on LCTS concept in one or two dimensions.

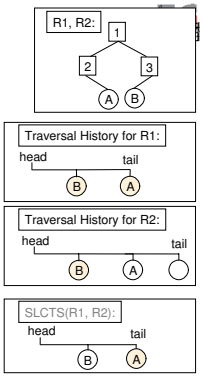


Other schemes: hierarchical image sampling

Ray Shooting 32

Simple LCTS = Sequence of Leaves



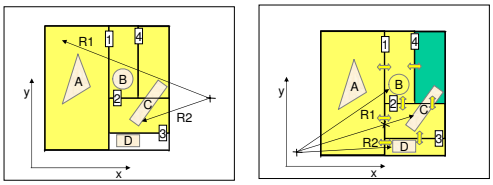


Ray Shooting 33

Simple LCTS - Problems

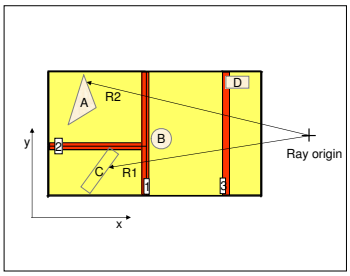
1) No common sequence of leaves exists.

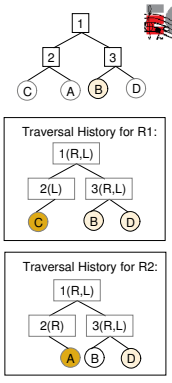
2) When accessing SLCTS, object was not found, and traversal has to continue further.



Ray Shooting 34

Hierarchical LCTS

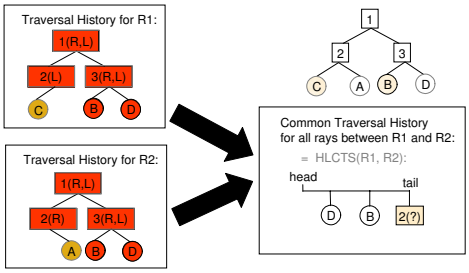




Ray Shooting 35

Hierarchical LCTS contd.

Matching two traversal histories into common one:

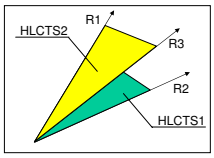


Ray Shooting 36

Hierarchical LCTS contd.

1) Matching traversal histories for two or more rays.

2) Matching traversal histories for rays with the previously constructed common traversal history.



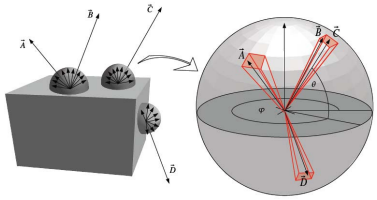
HLCTS1 - constructed from traversal history of R1 and R2

Ray R3 - traversal uses HLCTS1

HLCTS2 - constructed from HLCTS1 and traversal history of R3

Ray Cache in Final Gathering

- Store the rays into cache according to direction
- When a bucket is filled in, shoot all of them at once
- Improves access pattern for incoherent queries
- Speedup up to 30%



Surveys on Ray Shooting and Ray Tracing

- G. Simiakakis: Accelerating Ray Tracing with Directional Subdivision and Parallel Processing, 1995
- V. Havran: Heuristics Ray Shooting Algorithms, 2001
- I. Wald: Real Time Ray Tracing and Global Illumination, 2004
- A. Y-H. Chang: Theoretical and Experimental Aspects of Ray Shooting, 2005

Questions and Answers for Part One

Part 2 - Content

- Hidden Surface Removal
- Visibility Culling
- Photon Maps and Irradiance Caching
- Ray Maps
- Other Algorithms
- Questions and Answers (10 minutes)

1

Hidden Surface Removal

Jiří Bittner

Czech Technical University in Prague
Vienna University of Technology

2

Hidden Surface Removal

Find visible surface for every pixel (ray)

Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	**{rays}**	**{objects}**	**{points}**
Visibility culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Hidden Surface Removal

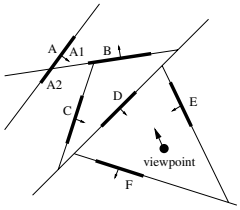
- List priority algorithms
- Area subdivision algorithms
- Scan-line algorithms
- Z-buffer
- Ray casting

Depth Sort

- Draw faces back to front [Newell72]
- Overwrite the farther ones (painter's alg.)
- Determine strict depth order
  - Resolve cycles of overlaping polygons
- Step 1: depth sort (Z)
  - Quick sort, bubble-sort (temporal coherence)
- Step 2: rasterization (YX)
  - Bucket sort to pixels

Depth Sort with BSP Tree

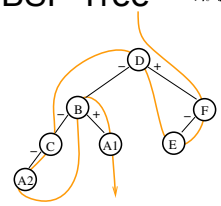
- BSP built in preprocess
  - Select a plane
  - Partition the polygons in front/back fragments
  - If >1 polygon → recurse



- Quick-sort like
- Heuristics for splitting-plane selection

Depth Sort with BSP Tree

- Tree size: $O(n^2)$
- Run-time: simple traversal

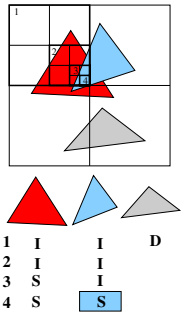


- Improvements
  - BSP need not be autopartition!
  - For manifolds depth order can be predetermined → coarser BSP
  - Generalization to all BSP nodes 'Feudal priority tree' [Chen96]

order: F,E,D,C,A2,B,A1

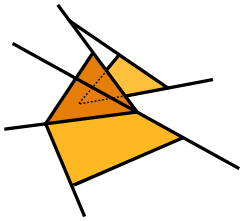
Area Subdivision

- Subdivide screen space [Warnock69]
- Classify polygons with respect to the area
- Terminate if trivial solution
- Step 1: octree subdivision (XY)
  - Quick sort like
- Step 2: list for octree nodes (Z)
  - Insertion sort



Naylor's BSP projection

- Draw polygons front to back
- Clip polygons by 2D BSP of projected polygons
- Step 1: depth sort (Z)
  - 3D BSP built in preprocess
- Step 2: 2D BSP (XY)
  - Quick sort like subdivision of the projection plane

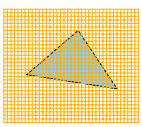


Scan-Line

- Sort by scan-lines (Y)
- Sort spans within a scanline (X)
- Search for closest span (Z)
- [Watkins70]
  - Bubble sort in X and Y
  - O(log n) search in Z

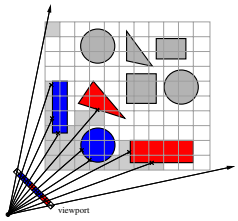
Z-buffer

- Rasterize polygons in arbitrary order
- Maintain per pixel depths
- Step 1: rasterization (YX)
  - Bucket sort like
- Step 2: per pixel depth comparison (Z)
  - Min selection



Ray Casting

- Cast ray for each pixel
- Step 1: spatial data structure (XYZ)
  - Preprocess
  - Trees ~ quick sort
  - Grid ~ distribution sort
- Step 2: search for nearest intersection
  - Min selection with early termination



Z-buffer vs. Ray Casting

	scan-line coherence	presorting	output sensitive
Z-buffer	yes +	no +	no -
Ray casting	no -	yes -	yes +

- Z-buffer better in simple sparsely occluded dynamic scenes
- Ray casting better in complex densely occluded static scenes

Summary

- HSR
  - Search for closest object for every pixel (ray)
- HSR algorithms sort in
  - Directions (XY)
  - Depth (Z)
  - Differ in sorting order and methods [Suth74]
- Current winners: z-buffer, ray casting

Visibility Culling

Jiří Bittner

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Vienna University of Technology

Visibility Culling

Find visible objects for a given view point or view cell

Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	**{rays}**	**{objects}**	**{objects}**
Photon maps	point	{points}	{points}
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Visibility Culling – Motivation

- Q: Why visibility culling, when:
  - Object outside screen culled by HW clipping
  - Occluded objects culled by z-buffer
- A: Linear complexity not sufficient!
  - Processing too many invisible polygons
- Goal
  - Render only what can be seen!
  - Make z-buffer output sensitive

Visibility Culling - Introduction

- Online
  - Applied for every view point at runtime
- Offline
  - Partition view space into view cells
  - Compute Potentially Visible Sets (PVS)

Online Visibility Culling

- For every frame cull whole groups of invisible polygons
- Conservative solution
  - Determines a superset of visible polygons
  - Precise visibility solved by z-buffer

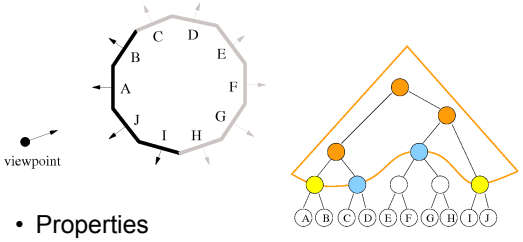
Online Visibility Culling

- View-frustum culling
- Occlusion culling
  - CPU techniques
  - GPU based (HW occlussion queries)

Backface Culling

- Culls about 50% polygons
- Supported by the GPU
- Alternative: Hierarchical back-face culling [Kummar96]
  - Sort polygons based on their normals into a tree
  - Skip whole groups of backfacing polygons

Hierarchical Backface Culling

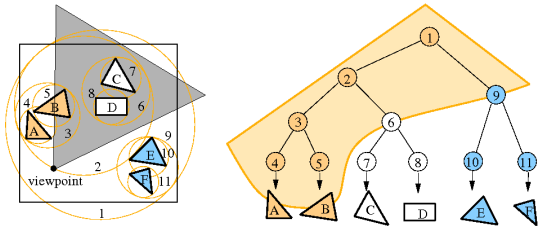


- Properties
  - \+ Skips whole groups of polygons
  - \- Regroups the scene (discards objects)
  - \- Limited gain

View Frustum Culling

- Culls 0-100% polygons
- Objects intersecting the view frustum
- Hierarchical VFC
  - Spatial hierarchy: kD-tree, BSP tree, octree, BVH
  - Intersection tests on hierarchy nodes
- Optimizations
  - Temporal coherence
  - Efficient intersection test [Assarson00]

View Frustum Culling - Example



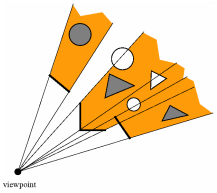
Occlusion Culling

- VFC disregards occlusion
- 99% of scene can be occluded!

- Solution: Detect and cull also occluded objects

Shadow Frusta

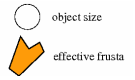
- Construct shadow frusta for several occluders [Hudson97]

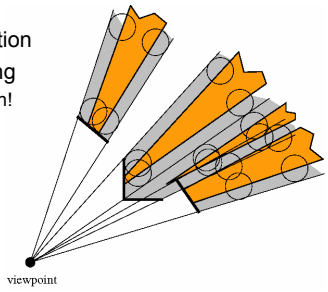


- Object is invisible if inside a shadow frustum
- Queries on the spatial hierarchy

Shadow Frusta - Properties

- Properties
  - + Easy implementation
  - - No occluder sorting
    - - No occluder fusion!
    - - O(n) query time
  - - Small number of occluders

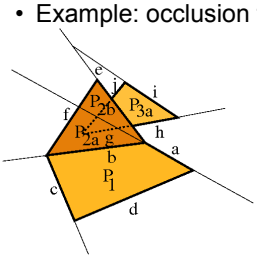


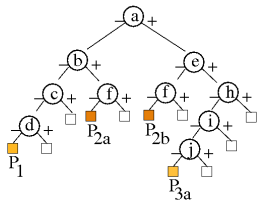


Visibility Culling 13

Occlusion Trees

- Occluders sorted into a 2D BSP tree [Bitt98]
- Occlusion tree represents fused occlusion
- Example: occlusion tree for 3 occluders

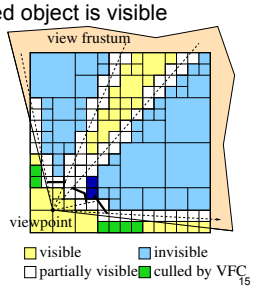




Visibility Culling 14

Occlusion Tree - Traversal

- Visibility test of a node
  - – Depth-first-search
  - – Found empty leaf → tested object is visible
  - – Depth test in filled leaves
- Example of final visibility classification of kD-tree



Visibility Culling 15

Occlusion Tree - Properties

- Presorting occluders
  - – Tree size: worst case $O(n^2)$, n = #occluders
  - – O(log n) visibility test
- + Allows to use more occluders (~100)
- - Not usable for scenes with small polygons

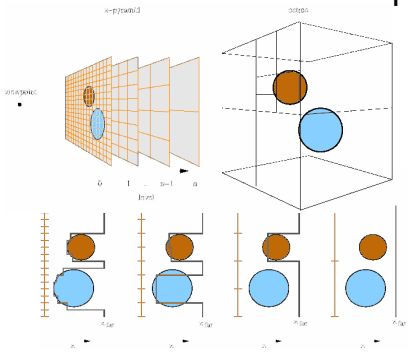
Visibility Culling 16

Hierarchical Z-buffer

- Extension of z-buffer to quickly cull larger objects [Greene 96]
- Ideas
  - – octree for spatial scene sorting
  - – z-pyramid for accelerated depth test

Visibility Culling 17

Hierarchical Z-buffer - Example



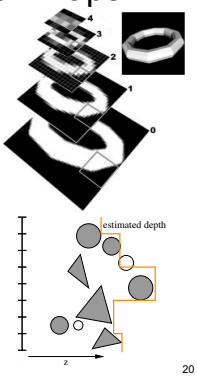
Visibility Culling 18

Hierarchical Z-buffer - Usage

- Hierarchical test for octree nodes
- Find smallest node of z-pyramid, which contains the tested box
- Box depth > node depth → cull
- Otherwise: recurse to lower z-pyramid level
- Optimization: use temporal coherence
  - z-pyramid constructed from polygons visible in the last frame

Hierarchal Occlusion Maps

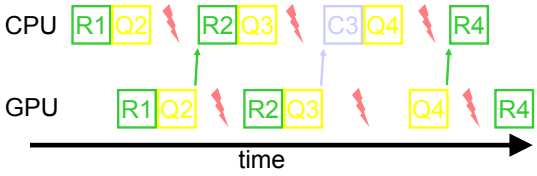
- Hierarchical occlusion maps [Zhang97]
- Pyramid of occlusion maps
- Separate occlusion and depth representation
  - Hierarchical occlusion
  - Coarse depth
- Queries on spatial hierarchy



HW Occlusion Queries

- ARB_occlusion_query, NV_occlusion_query
- Return #pixels passing the depth test
- Feature which has been missing!
- Issues
  1. Latency – the result not readily available
  2. The query costs time

CPU Stalls GPU Starvation

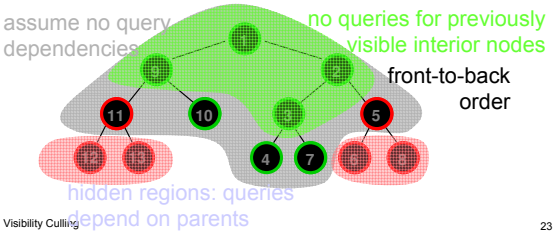


Rx Render object x
Qx Query object x
Cx Cull object x

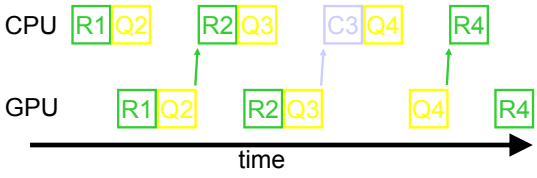
Waiting time

Coherent Hierarchical Culling

- CHC [Bitt04]
  - Interleave queries and rendering
  - Schedule queries based on temporal coherence



CHC



Rx Render object x
Qx Query object x
Cx Cull object x

CHC

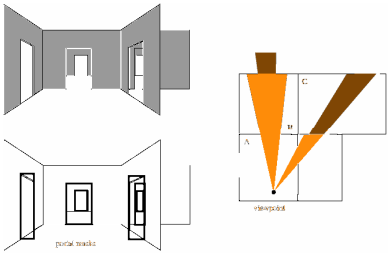
- Video
- UNC power plant, 12.7M polygons

Cells and Portals

- Partition the scene in cells and portals
  - Cells ~rooms
  - Portals ~ doors&windows
- Cell adjacency graph
- Constrained search
  - Portal visibility test [Luebke 96]

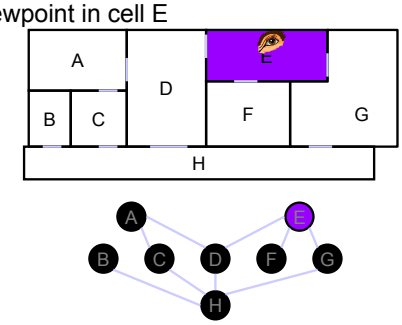
Portal Visibility Test

- Intersection of bounding rectangles of portals



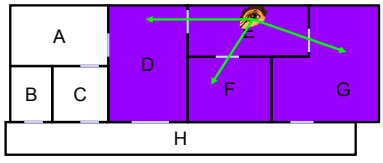
Cells and Portals Example

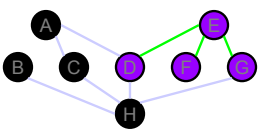
- Viewpoint in cell E



Cells and Portals - Example

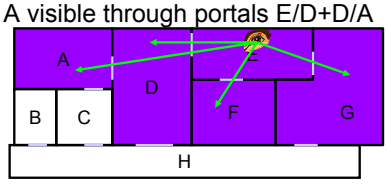
- Adjacent cells DFG

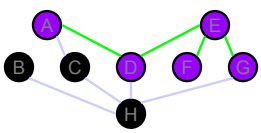




Cells and Portals - Example

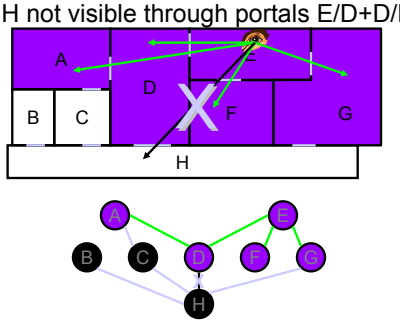
- Cell A visible through portals E/D+D/A





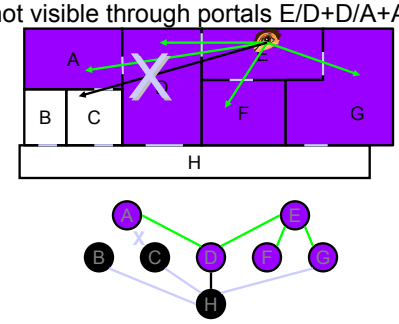
Cells and Portals - Example

- Cell H not visible through portals E/D+D/H



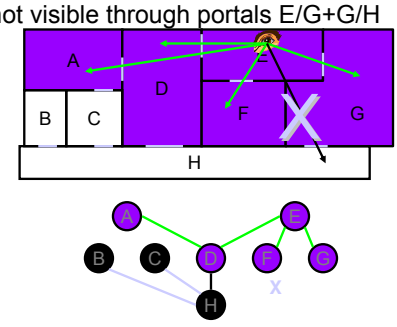
Cells and Portals - Example

- C not visible through portals E/D+D/A+A/C



Cells and Portals - Example

- H not visible through portals E/G+G/H



Visibility Preprocessing

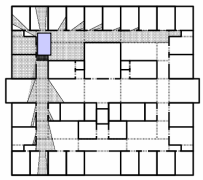
- Preprocessing
  - Subdivide view space into view cells
  - Compute Potentially Visible Sets (PVS)
  - Solves visibility “offline” for all possible view points
- Usage
  1. Find the view cell (point location)
  2. Render the associated PVS

Visibility Preprocessing

- Other benefits
  - Prefetching for out-of-core/network walkthroughs
  - Communication in multi-user environments
- Problems
  - Costly computation (treats all view points and view directions)
  - PVS storage

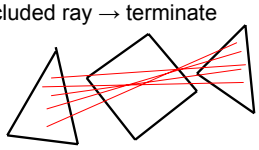
Interiors – Cells and Portals

- Subdivide the scene into cells and portals
- Constrained DFS on the adjacency graph
  - Portal visibility test
- More complex than the online algorithm
  - We do not have a view point!



Interiors – Cells and Portals

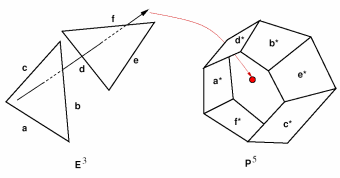
- Sampling [Airey90]
  - Random rays
  - Non-occluded ray → terminate



+ Simple implementation
- Approximate solution

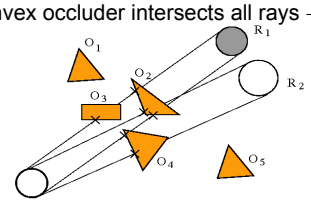
Interiors – Cells and Portals

- Exact computation [Teller 92]
  - Mapping to 5D (Plücker coordinates of lines)
- Portal edges → hyperplanes H_i in 5D
- Halfspace intersection in 5D



General Scenes - Strong Occlusion

- Occlusion by single object [CohenOr98]
- For each cell and object
  - Cast rays defining convex hull of the cell and object
  - If a convex occluder intersects all rays → invisible



General Scenes - Strong Occlusion

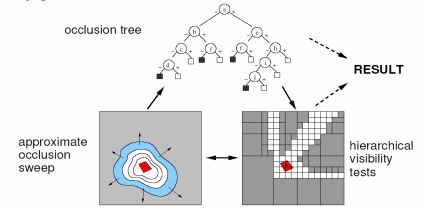
- Properties
  + Simple
  - No occluder fusion (no occluder sorting)
  - Too conservative for larger view cells and small objects

General Scenes Occlusion Tree

- Extension of the 2D occlusion tree
- 5D BSP tree
  - Plücker coordinates of lines
- The tree represents union of occluded rays

General Scenes Occlusion Tree

- Process polygons in front-to-back order
- Polygon visible → enlarge tree by visible rays
- Polygon invisible → tree not modified

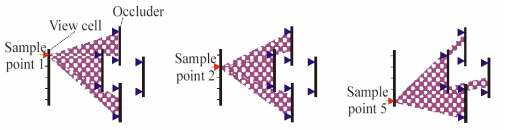


General Scenes
Occlusion Tree

- Properties
  - + Exact solution
  - + Uses visibility coherence
  - - Difficult implementation

2.5D Scenes
Occluder Shadows

- Footprint of occluded volume [Wonka00]
  - – Agrregates the shadow polygons using z-buffer
  - – Represents intersection of all ‘shadows’



2.5D Scenes
Occluder Shadows

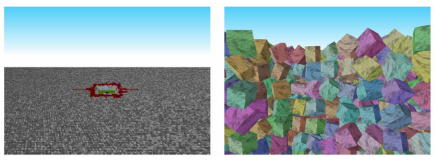
- Conservative solution
  - – Shrinking occluder polygons
- Properties
  - + Relatively easy implementation
  - + Uses GPU
  - - Large view cells → more conservative solution
  - - Needs high resolution cull map

2.5D Scenes
Ray Space Factorization

- Main ideas [Leyvand et al. 2003]
  - – Occluder in 2.5D → 3D polygon in ray space
  - – Polygon shape: defined by 2D footprint
  - – Polygon depth: defined by heights

2.5D Scenes
Ray Space Factorization

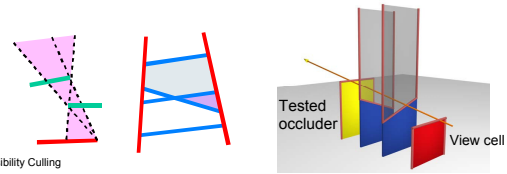
- Render polygons using z-buffer
- Visible polygons in ray space → visible objects in primal space



- Properties
  - - Conservative solution
  - + GPU implementation

2.5D Scenes
Occlusion Tree + Virtual Portals

- Occlusion tree for visibility in 2D footprint
- Identifies sequencies of occluders
- Construct virtual portals over the occluders
- Portal visibility test in 5D [Teller 92]



2.5D Scenes
Occlusion Tree + Virtual Portals

- Properties
  - + exact solution for 2.5D scenes
  - + computation time comparable with conservative methods
  - - difficult implementation

Visibility Culling - Summary

- Find visible objects for a view point or view cell
- Heavy use of spatial sorting
  - – Common HDS: kD-tree, octree, BVH
- Occlusion culling differs in occluder sorting
  - – No sorting, occlussion trees, HOM, cells + portals
- Online vs. offline culling
  - – Online: dynamic scenes
  - – Offline: very fast at runtime for static scenes

Surveys on Visibility

- F. Durand. 3D Visibility: Analytical Study and Applications, 1999.
- D. Cohen-Or et al.: A survey of visibility for walkthrough applications, 2003.
- J. Bittner and P. Wonka: Visibility in computer graphics, 2003.

Photon Maps and Irradiance Caching

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Photon Maps and Irradiance Caching

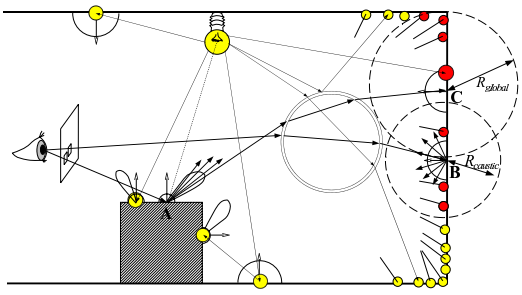
- Final gathering versus direct visualization
- Photon maps
- Irradiance caching
- Offline techniques

Photon Maps

Find nearest photons given a point

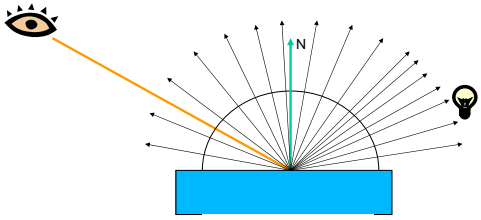
Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	{rays}	{objects}	{objects}
Photon maps	**point**	**{points}**	**{points}**
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Photon Mapping



Final Gathering

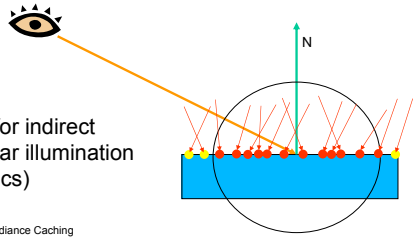
- Shooting many secondary rays (possibly according to BRDF), gathering radiances from the rays
- Integrating the radiances properly to render image



- Used for indirect diffuse illumination

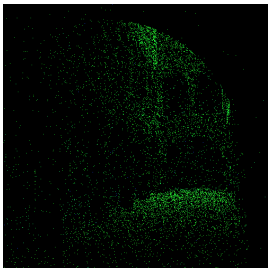
Direct Visualization

- Do not shoot final gather rays, use directly visible photons from camera
- It is prone to artifacts on object boundaries referred to as bias



- Used for indirect specular illumination (caustics)

Example of Direct Visualization



Photon Hits

Direct Visualization

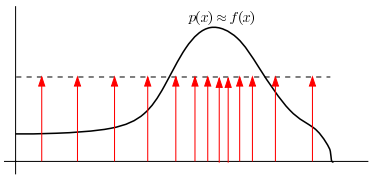
Estimating Radiance along Final Gather Ray

- Using the density estimation, from the photon hits estimating *PDF*
- It requires K nearest neighbor searching for each final gather ray
- The number of final gather rays (the number of searches) is enormous
- Typically we shoot 200-4000 final gather rays per pixel
- The number of pixels per image 1-6 x 10^6

Intro to Density Estimation

- Histogram method – take hits into buckets
- Kernel density estimation
- K-Nearest neighbors estimator
- Variable kernel density estimator
- Multiple pass methods
  - First pass – pilot estimate
  - Second pass – final estimate

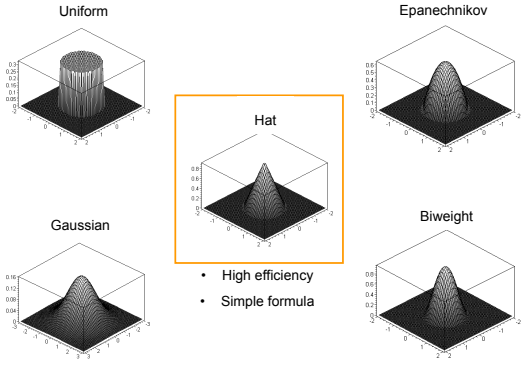
Example: Density Estimation in1D



Note: **Importance Sampling**: from given *p(x)* to samples

Density Estimation: from samples to *p(x)* … more complicated

Kernel Types



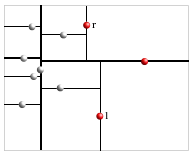
- High efficiency
- Simple formula

Relation to Searching

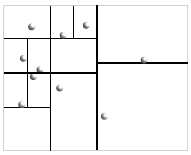
- *Range search* – given a fixed range query (sphere, ellipsoid), find all the photons in the range
- *K nearest neighbor search* – given a center of the expanding shape X (sphere, ellipsoid), find K nearest photons
  - Without considering the direction of incoming photons
  - With considering only valid photons with respect to the normal at point X

Search Techniques

- Use any data structures described in the section "*Hierarchical Data Structures*"
- Typically kd-trees or kd-B-trees are used



Kd-tree

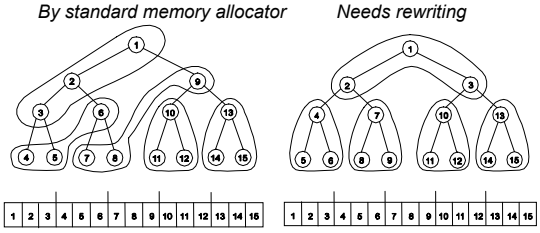


Kd-B-tree

Data Layout in Memory

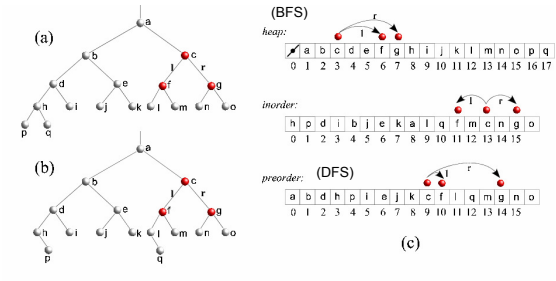
Inorder, preorder (depth-first-search), heap (bread-first-search), van Emde Boas

By standard memory allocator *Needs rewriting*



Depth-first-search (DFS) Van Emde Boas

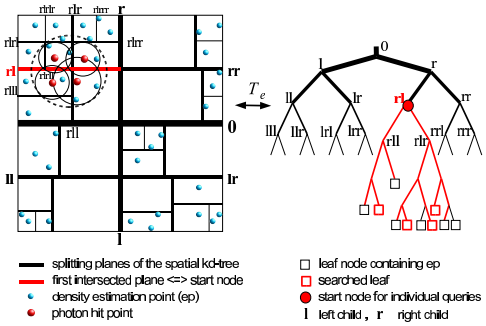
KD-tree Layout in Memory



Practical yet Efficient Solution

- Use Kd-B-trees
- Construct a tree over an array of photons
- Use 8 Bytes nodes – good packing
- DFS or van Emde Boas Layout
- Sliding mid-point rule = spatial median + shift to nearest photon if one side empty
- One leaf contains a range of 30-70 photons (two indices to photon array)
- Properties:
  - fast construction time
  - fast search (complexity proved to be optimal)

Aggregate Searching (= Offline Search)



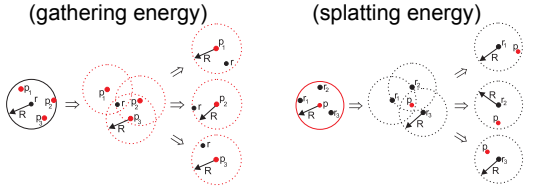
Searching Tricks for k-NN Search

- Do not use uniform grids, they do not work efficiently for skewed distributions
- Try to avoid a priority queue by using a fixed radius search, where the radius is estimated for given N (from already computed queries or the diagonal of a leaf box)
- Use offline search if possible
- Try to change the role of input data to be queried and queries

Reverse Photon Mapping

Normal Photon Mapping (gathering energy)

Reverse Photon Mapping (splatting energy)



r – ends of final gather rays (in black)

p – photons (in red)

Why Does It Work Faster ?

Assume that the number of interactions among photons and final gather rays is the same !

Traditional Photon Mapping – a single tree

- Many searches (~10^9) in a small tree over photons (~10^6)
- kNN search based on the photon density

Reverse Photon Mapping – more involved *(two trees)*

- Smaller number of searches (~10^6) in a larger tree over the ends of final gather rays (~up to 10^9)
- k-NN search is also based on the photon density

Properties

- Search in a tree is logarithmic, reverse photon mapping then faster
- Reverse photon mapping takes more memory

Time Complexity Formulas

- F … number of final gather rays
- K … number of neighbors for kNN search
- V … number of photons
- F * K … number of interactions photon-final gather ray

Traditional Photon Mapping Time:

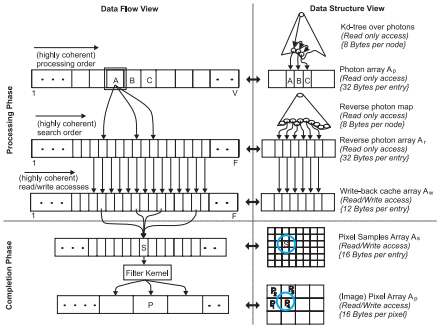
C_PT = C_1 * F * K + C_2 * F * log V

Reverse Photon Mapping Time:

C_RPT = C_1 * F * K + C_2 * V * log F

For **F >> V** it is easy to show that **F * log V > V * log F**

Data Flow + Data Structure View

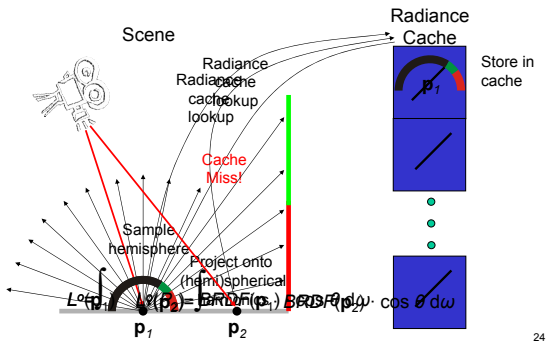


Irradiance Caching

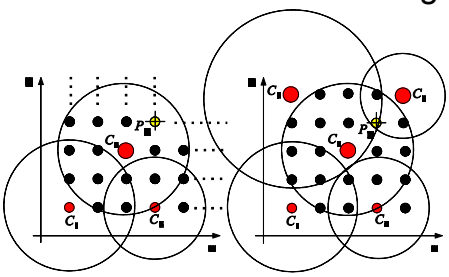
Find all spheres containing a given point

Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	point	{rays}	{rays}
Irradiance caching	point	{spheres}	{spheres}

Radiance and Irradiance Caching



Data Structures for Caching

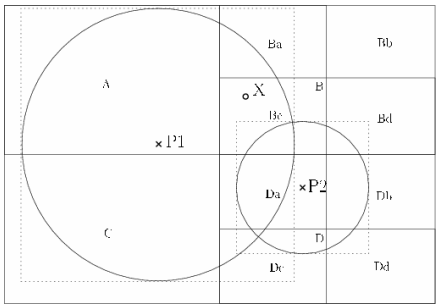


- Red – data in the cache
- Black – queries

Search Specification

- Records – the irradiance specified by a point and radius of influence
- Query: given a point, find all the sphere in which the point is contained
- Problem is *intersection search*
- Data structures should be dynamic – insertion and deletion is possible

1) Using Octree (Ward et al. 88)



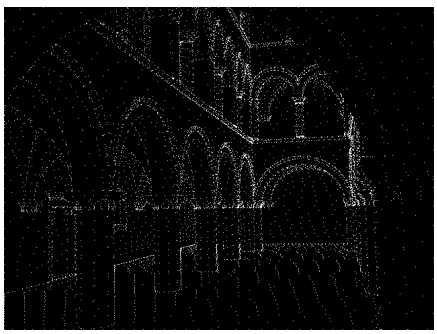
2) Using Mapping to R^4

- A sphere (a,b,c,r) in R^3 as a point in R^4 (t1,t2,t3,t4) by linearization:

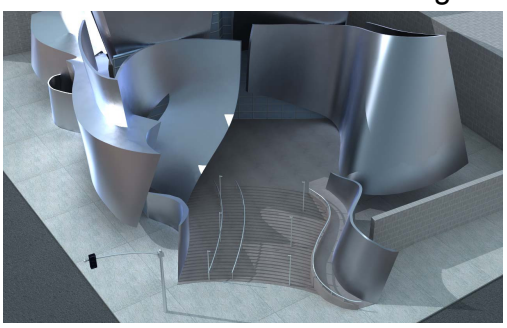
  (2.a, 2.b, 2.c, a*a + b*b + c*c - r*r)
- Query: a point (a, b, c) in R^3 as a hyper-plane in R^4 (t1,t2,t3,t4) as follows:

  H: a*t1+b*t2+c*t3-t4 –(a*a+b*b+c*c) > 0
- Compute *half-space range reporting* in R^4 space, it requires a spatial data structure in R^4
- Efficiency depends highly on
  - Position of points with respect to the space origin
  - Efficiency of half-space range reporting

Irradiance Caching Records



Positions of Samples in Radiance Cache – Final Image



Ray Maps

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Ray Maps

Problem: Find nearest rays for a given point

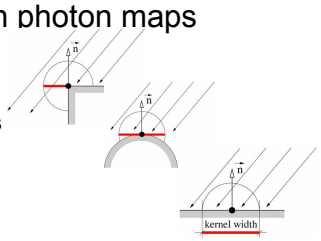
Problem	Q	S	A
Ray shooting	ray	{objects}	point
Hidden Surface Removal	{rays}	{objects}	{points}
Visibility culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	**point**	**{rays}**	**{rays}**
Irradiance caching	point	{spheres}	{spheres}

Ray Maps

- Ray map: data structure sorting rays
- Allows efficient searching for rays
  - nearest to a point (k-NN)
  - intersecting a disc/sphere/hemisphere
- Main application:
  improved density estimation

Density Estimation

- Problems with photon maps
  1. Boundary bias
  2. Topological bias
  3. Proximity bias



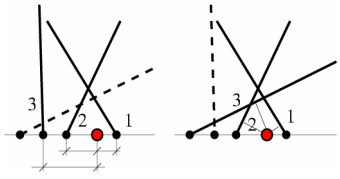
- Ray maps
  eliminate 1. and reduce 2.

Ray Map Queries

- Queries types
  - intersection search
  - k-NN search
- Query domains
  - disc
  - sphere
  - hemisphere
  - axis-aligned box
  - + possible limitation on ray directions

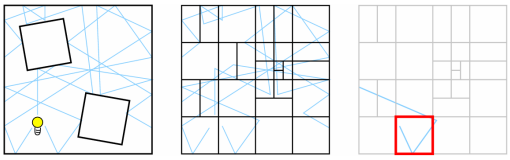
Metrics for k-NN Search

- Distance on the tangent plane
- Distance to the ray segment
- Distance to the supporting line of the ray



Ray Map Implementation

- Kd-tree
- Leaves store references to the rays
- Lazy construction driven by the queries
- Support efficient searching and updating



Construction

- Spatial median split
- Subdivide if #rays > budget
- Classify rays back, front, both
- Termination criteria
  - #ray references per leaf (~32)
  - size of the leaf (~0.1% of the scene box)
  - max tree depth (~30)

Searching

- Intersection search
  - Locate all leaves containing query domain
  - Gather rays
  - Compute intersections
- k-NN search
  - Priority queue
  - Locate the leaf containing the query origin
  - If #rays < N get next node from the queue

Maintenance

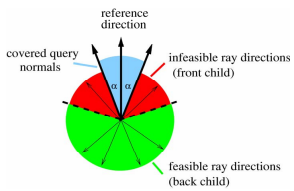
- Deleting a ray
  - Ray cast and remove references
- Adding a ray
  - Ray cast and subdivide if required
- Keeping memory budget
  - Collapsing of unused subtree nodes
  - Least-recently-used strategy

Optimization 1
Exploiting Query Coherence

- Subsequent queries often coherent
- Store traversed nodes of the previous query
- Initiate the priority queue with the saved nodes
- Top-down traversal is reduced

Optimization 2
Directional Splits

- Queries are oriented
- Many rays in the opposite direction after reflection
- Optimization: inserting directional nodes

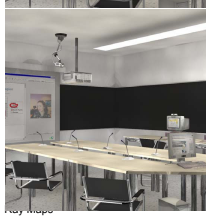


k-NN Search with Ray Maps

- 1M – 2.5M rays
- Typical memory usage: 16 – 128MB
- Query time (500-NN): 0.2–1.5ms (3.2GHz PC)
- ~ 2 - 5 times slower than photon maps

Comparison with Photon Maps

Photon maps



Photon maps + convex hull

Ray maps

1,000,000 photons, 500-NN

Similar Data Structures

- Ray cache [Lastra02]
  - Hierarchy of spheres
- Volumetric ray density estimation [VanHaevre04]
  - Octree
  - Simulation of plant growth

Ray Maps in Line Space

- Idea
  - Ray → 5D point (Plücker coordinates)
  - Query → 5D polyhedron
  - Report all points in the polyhedron
  - Use 5D kD-tree to sort points
- Poor performance
  - Culling only at very bottom of the tree
  - 5D boxes not separate well from the query polyhedron

Ray Maps - Summary

- Sorting rays + efficient searching
- Kd-tree implementation
  - \+ Simple implementation
  - \+ Efficient memory usage control
- Density estimation
  - \+ New query domains + new metrics
  - \+ Elimination of boundary bias
  - \+ Reduction of topological bias
  - \- 2-5x slower than photon maps

Other Algorithms

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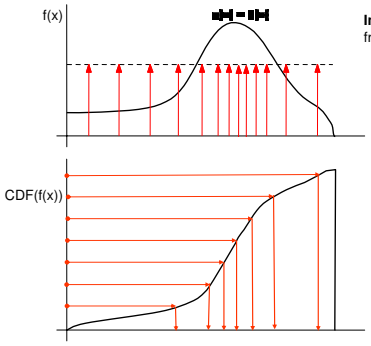
Other Algorithms

- Importance sampling
- Hierarchies over light sources
- Extensions to ray tracing
- Some other techniques

Note: this list on sorting and searching in rendering is definitely not complete !

Importance Sampling



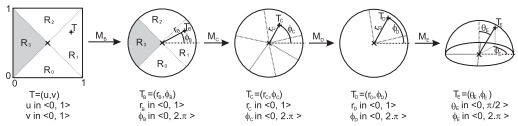
Importance Sampling: from given *p(x)* to samples

Step 1: compute cumulative distribution function into a table

Step 2: via uniform distribution over CDF generate required distribution f(x)

Importance Sampling Transforms

- *Results:* samples on the hemisphere (2D domain)
- *Usage:* for BRDF and environment maps
- *Realization:* using four mappings
- *Properties:* bijectivity, continuity in both directions, low distortion
- *Complexity of sampling:* O((log N) * (log M))



Havran et al. 03c: Goniometric Diagram Mapping for Hemisphere

Hierarchies over Light Sources

- Another hierarchy (=sorting) if number of light sources is high, approximating or discarding less important light sources
- Papers:
  - Ward92: Adaptive Shadow Testing, discard less important contributions, avoid shadow rays testing
  - Lazanyi and Szirmay-Kalos 04: Speeding up the Virtual Light Sources Algorithm
  - Paquette et al. 98: A Light Hierarchy for Fast Rendering of Scenes with Many Lights
  - Walter et a. 05: Lightcuts: a scalable approach to illumination
  - Walter et al. 06: Multidimensional lightcuts

Extensions to Ray Tracing

- Spatio-temporal domain
  - Continuous setting (Glassner 88)
  - Multiframe ray tracing (discrete time setting) (Havran et al. 03b)
  - Reprojection for walkthroughs (Havran et al. 03a)
- Approximate ray tracing
  - Szirmay-Kalos et al.: Approximate Ray-Tracing on the GPU with Distance Impostors (2005)
- Fast construction or update for animations
  - Several algorithms proposed in 2005, not yet resolved issue

Some Other Techniques

- Temporal Photon Mapping and Spatio-Temporal Density Estimation
  - Cammarano and Jensen 02: Time Dependent Photon Mapping
  - Weber et al. 2004: Spatio-Temporal Photon Density Estimation Using Bilateral Filtering
- Reordering the queries for photon mapping
  - Havran et al. 05: Reverse Photon Mapping
  - Steinhurst et al. 05: Reordering for Cache Conscious Photon Mapping
- Changing the role of queries and input data to be queried
  - Havran et al. 05: Reverse Photon Mapping (here in the slides)
  - Laine and Aila 05: Hierarchical Penumbra Casting

Remainder and Question

- This list on the use of sorting and searching in rendering algorithms is definitely not complete!
- Are you convinced now that sorting and searching is really relevant to rendering?

Content

Part One

- Introduction to Rendering
- Sorting and Searching
- Hierarchical Data Structures
- Ray Shooting
- Questions and Answers

Content

Part Two

- Hidden Surface Removal
- Visibility Culling
- Photon Maps and Irradiance Caching
- Ray Maps
- Other Algorithms
- Questions and Answers

Questions and Answers for Part Two

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