

Advanced Drawing Beautification with ShipShape

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Abstract

Sketching is one of the simplest ways to visualize ideas. Its key advantage is its easy availability and accessibility, as it requires the user to have neither deep knowledge of a particular drawing program nor any advanced drawing skills. In practice, however, all these skills become necessary to improve the visual fidelity of the resulting drawing. In this paper, we present ShipShape—a general beautification assistant that allows users to maintain the simplicity and speed of freehand sketching while still taking into account implicit geometric relations to automatically rectify the output image. In contrast to previous approaches ShipShape works with general Bézier curves, enables undo/redo operations, is scale independent, and is fully integrated into Adobe Illustrator. We show various results to demonstrate the capabilities of the proposed method (Figure 1).

Keywords: Drawing system, Input beautification, Vector graphics, Visual feedback

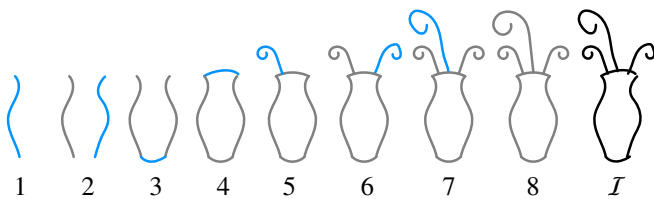


Figure 2: Incremental beautification workflow. Every newly drawn stroke (blue) is beautified using previously created data (gray). The first stroke is left unchanged. As the drawing continues, more suitable geometric constraints emerge and are applied, such as path identity (2,6,7), reflection (2,6) or arc fitting (3,4). For comparison with the final beautified output (8), I shows the original input strokes.

1. Introduction

Sketching with a mouse, tablet, or touch screen is an easy and understandable way to create digital content, as it closely mimics its real-world counterpart, pen and paper. Its low demands make it widely accessible to novices and inexperienced users. However, its imprecision means that it is usually only used as a preliminary draft or a concept sketch. Making a more polished drawing requires significantly more time and experience with the drawing application being used. Furthermore, when working with drawing or sketching software, users are often forced to switch between different drawing modes or tools or to memorize cumbersome shortcut combinations.

While we do not question the necessity or usefulness of complex tools to achieve non-trivial results, we argue that for certain scenarios, such as geometric diagram design or logo study creation, the *interactive beautification* [1] approach is more beneficial. Such workflows retain the intuitiveness of freehand input while benefiting from an underlying algorithm that automatically rectifies strokes based upon their geometric relations, giving them more formal appearance. With the

quickly growing popularity of touch-enabled devices, the applicability of this approach expands greatly. However, whatever the potential of automatic beautification in a more general sketching context, most of the existing applications focus on highly structured drawings like technical sketches.

One of the biggest challenges in drawing beautification is resolving ambiguity of the user input, since the intention and its execution are often considerably dissimilar. Additionally, this issue becomes progressively more complex as the number of primitives present in the drawing increases.

In this paper, we present a system for beautifying freehand sketches that provides multiple suggestions in spirit of Igarashi et al. [1]. Strokes are processed incrementally (see Figure 2) to prevent the combinatorial explosion of possible outputs. Unlike previous work, our approach supports polycurves composed of general cubic Bézier curves in addition to simple line segments and arcs. The system is scale-independent, and can easily be extended by new operations and inferred geometric constraints that are quickly evaluated and applied. The algorithm was integrated into Adobe Illustrator, including undo/redo capability. We present various examples to demonstrate its practical usability.

2. Related Work

The need to create diagrams and technical drawings that satisfy various geometric constraints led to the development of complex design tools such as CAD systems. However, these systems' complexity often limits their intuitiveness. Pavlidis and Van Wyk [2] were one of the first to try to alleviate this conflict by proposing a method for basic rectification of simple rectangular diagrams and flowcharts. However, their process became ambiguous and prone to errors when more complex

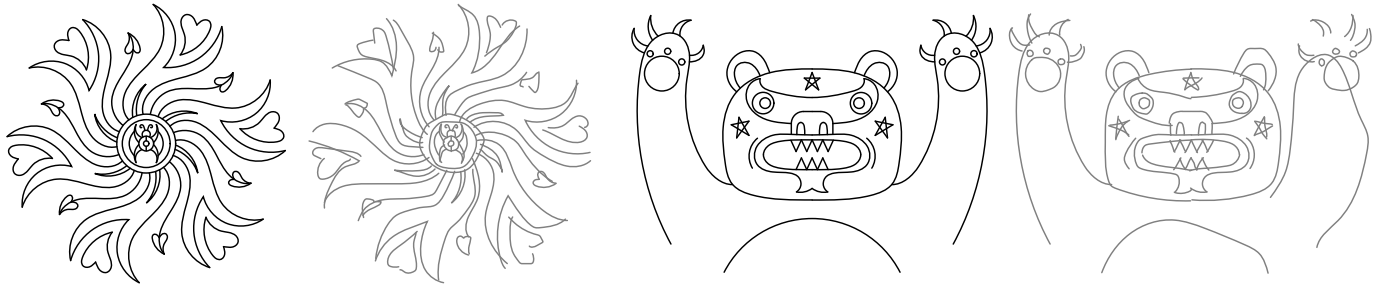


Figure 1: Examples of drawings created using ShipShape. The final drawings (black) were created from the imprecise user input (gray) by beautifying one stroke at a time, using geometric properties such as symmetry and path identity. See Figure 17 for more results.

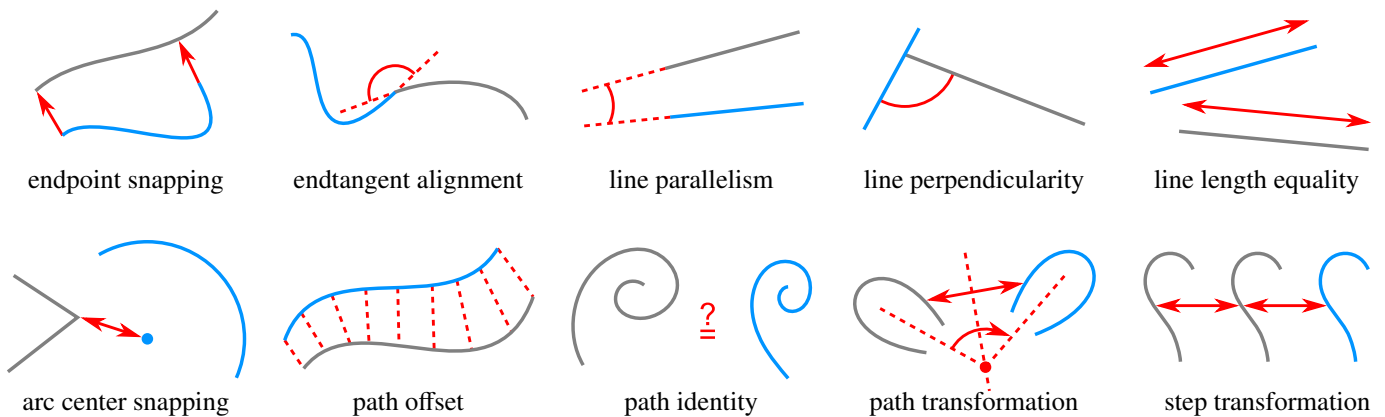


Figure 3: Supported geometric rules and transformations in our framework. The blue paths represent the data being beautified, while gray paths are data already processed. For more detailed description of the criteria used to evaluate these constraints, see Section 3.1.

52 drawings were considered, since the method needed to drop
53 many constraints to keep the solution tractable.

54 To alleviate this limitation, Igarashi et al. [1] proposed an in-
55 teractive beautification system in which the user added strokes
56 one by one and the system improved the solution incrementally
57 while keeping the previously processed drawing unchanged.
58 This solution kept the problem tractable even for very complex
59 drawings. Moreover, the system also presented several beauti-
60 fied suggestions and let the user pick the final one. This brought
61 more user control to the whole beautification process. Follow-
62 ing a similar principle, other researchers developed systems for
63 more specific scenarios such as the interactive creation of 3D
64 drawings [3], block diagrams [4, 5], forms [6], and mathemat-
65 ical equations [7].

66 However, a common limitation of the approaches mentioned
67 above is that they treat the image as a set of line segments. To
68 alleviate this drawback Paulson and Hammond [8] proposed a
69 system called *PaleoSketch* that fit the user input to one of eight
70 predefined geometric shapes, such as line, spiral or helix. In
71 a similar vein, Murugappan et al. [9] and Cheema et al. [10]
72 allowed line segments, circles and arcs.

73 Related to drawing beautification, there are also approaches
74 to beautify curves independently, without considering more com-
75 plex geometric relationships. Those approaches are orthogonal
76 to our pipeline. They use either geometric curve fitting [11, 12]
77 or some example-based strategy [13, 14]. Additionally, ad-

78 vanced methods for vectorizing and refining raster inputs have
79 been proposed [15, 16], which enable users to convert bitmap
80 images into high quality vector output. However these do not
81 exploit inter-stroke relationships. In our case we assume that
82 the built-in curve beautification mechanism of Adobe Illustra-
83 tor preprocesses the user’s rough input strokes into smooth, fair
84 paths.

85 This paper extends our previous work [17]. In Section 3.1
86 we discuss improvements to the arc and circle center rules, and
87 introduce a generalized transformation adjustment framework.
88 Section 3.4 describes a new method for curve alignment, and
89 Section 3.5 describes the transformation adjustment mechanism
90 in detail. Finally, Section 4 describes a new framework for han-
91 dling curves with corners.

92 3. Our Approach

93 A key motivation for our system is wanting to work with ar-
94 bitrarily curved paths. This capability was not available in pre-
95 vious beautification systems. Although some can recognize a
96 variety of curves including spirals and general 5th degree poly-
97 nomials (*PaleoSketch* [8]), they recognize them only in isola-
98 tion and do not allow to take other existing paths into consid-
99 eration, which is important for interactive design.

100 Systems like that of Igarashi et al. [1] generate a set of po-
101 tential constraints and then produce suggestions by satisfying

102 subsets of these. A key challenge that prohibits simply gen-
 103 eralizing these systems to support general curved paths is the
 104 number of degrees of freedom, which boosts the number of po-
 105 tential constraints that need to be evaluated. Moreover, unlike
 106 line or arc segments, many of a general path’s properties, for
 107 example the exact coordinates of a point joining two smooth
 108 curves, do not have any meaning to the user. It would not be
 109 helpful to add constraints for this point. Finally, satisfying con-
 110 straints on a subset of the defining properties might distort the
 111 path into something that barely resembles the original. Sup-
 112 porting generalized paths requires a different approach.

113 Our system is based on an extensible set of self-contained
 114 geometric rules, each built as a black box and independent of
 115 other rules. Every rule represents a single geometric property,
 116 such as having an endpoint snapped or being a reflected ver-
 117 sion of an existing path. The input to each rule is an input path
 118 consisting of an end-to-end connected series of Bézier curves,
 119 and the set of existing, resolved paths. The black box evaluates
 120 the likelihood that the path conforms to the geometric property,
 121 considering the resolved paths, and outputs zero or more mod-
 122 ified versions of the path. Each modified version gets a score,
 123 representing the likelihood that the modification is correct.

124 For example, the same-line-length rule would, for input that
 125 is a line segment, create output versions that are the same lengths
 126 as existing line segments, along with scores that indicate how
 127 close the segment’s initial length was to the modified length.
 128 Each rule also has some threshold that determines that the score
 129 for a modification is too low, and in that case it does not output
 130 the path.

131 The rules also mark properties of the path that have become
 132 fixed and therefore can no longer be modified by future rules.
 133 For example, the endpoint-snapping rule marks one or both
 134 endpoint coordinates of a path as fixed. The same-line-length
 135 and parallel-line rules do not attempt to modify a segment with
 136 two fixed endpoints.

137 Since the rules do not depend on each other, it is easy to add
 138 new rules to support additional geometric traits. Figure 3 shows
 139 an illustrated list of rules supported in our system.

140 Chaining the rules can lead to complex modifications of the
 141 input stroke and is at the core of our framework. We treat the
 142 rule application as branching in a directed rooted tree of paths,
 143 where the root node corresponds to the unmodified input path.
 144 Each branch of the tree corresponds to a unique application of
 145 one rule and the branch is given a weight corresponding to the
 146 rule’s score.

147 To find suitable transformations for the user input, we tra-
 148 verse down to the leaf nodes (see Figure 4).

149 Formally, given a node n^i with Bézier path p^i , the set of
 150 resolved paths S , and the set of all rules $r_j \in R$, we compute an
 151 output set $P^i = \{r_j(p^i, S)\}$. We then create a child node n_j^i for
 152 each $p_j^i \in P^i$. If P^i is empty, n^i is a leaf node.

153 Since we need to compare scores among different rules,
 154 likelihoods are always normalized into the interval $[0, 1]$. If
 155 a rule generates any modified paths, it also generates a copy of
 156 the unmodified path, indicating the suggestion that the rule did
 157 not apply. The likelihood for the unmodified path is 1 minus

158 the maximum likelihood of any modified path.

We can then use all scores from the nodes we visited while
 descending into a particular leaf node n to calculate the overall
 likelihood score for the chained transformation as

$$\overline{\mathcal{L}}_i = 1 - \prod_{k=1}^{d-1} (1 - \mathcal{L}(r_j(a^k, S))) \quad (1)$$

159 where d is the depth of n in the tree, a^k is the k th ancestor of
 160 n , and $\mathcal{L}(r_j(a^k, S))$ denotes the likelihood score from applying
 161 rule r_j to node a^k .

162 We expand the search tree in a *best-first search* manner,
 163 where the order of visiting the child nodes is determined by the
 164 overall score $\overline{\mathcal{L}}$ of the node’s path. While traversing the tree,
 165 we construct a suggestion set Q of leaf nodes, which is initially
 166 empty and gets filled as the leaf nodes are encountered in the
 167 traversal. Once not empty, Q helps prune the search. Before we
 168 expand a particular subtree, we compare the geometric proper-
 169 ties of its root with properties of each path $q \in Q$. If all tested
 170 properties are found in some path q , the whole subtree can be
 171 omitted from further processing (see Figure 5).

172 Furthermore, to keep the user from having to go through
 173 too many suggestions, we limit the size of Q . Since we traverse
 174 the graph in a best-first manner, we stop the search after finding
 175 some number of unique leaf nodes (10 in our implementation).

176 3.1. Supported Rules and Operations

177 Geometric transformations in our framework are evaluated
 178 by testing various properties of the new path and the set of pre-

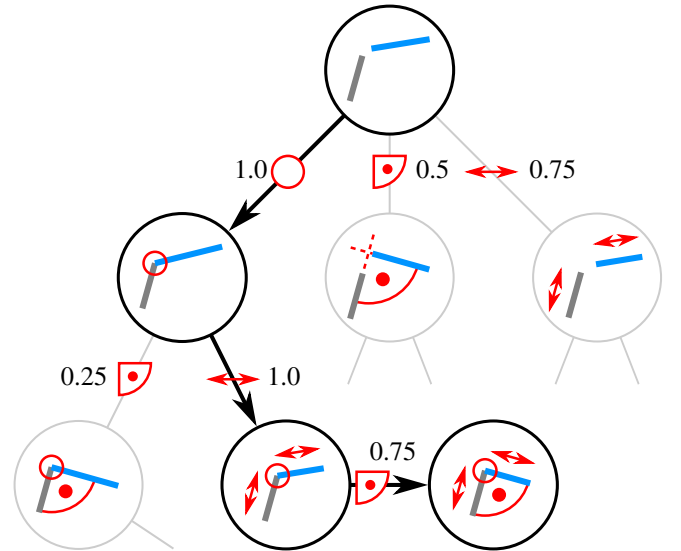


Figure 4: Successive rule evaluation and application. In this example, the eval-
 uation engine consists of three geometric rules—endpoint snapping, perpen-
 dicularity, and length equality. The old data (gray path) is fixed in the canvas.
 When a new path (blue) is added, it becomes the root node of the evaluation
 graph and the expansion begins by testing all rules on it. A likelihood score is
 calculated for each rule application and the tree is expanded using a best-first
 search scheme, until leaf nodes are reached. Due to the significant redundancy
 in the search space, many leaf nodes will contain duplicate suggestions. There-
 fore, we prune the graph during the expansion step using the information from
 already reached leaf nodes (see Section 3 and Figure 5 for more information).

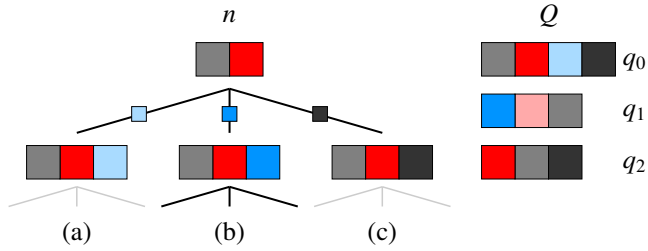


Figure 5: Search graph pruning. The rules are represented by colored boxes with hue being distinct rules and lightness their unique applications (e.g., if red color represents endpoint snapping, then different shades of red correspond to snapping to different positions). An inner node n has been expanded into three branches (a,b,c). Before further traversal, all subtrees stemming from the child nodes of n are tested against suggestions $q \in Q$. Here, branches (a) and (c) are fully contained in q_0 and q_2 respectively and thus only branch (b) is evaluated further.

179 viously drawn and processed paths. While tests of some prop-
 180 erties are simple, others, such as path matching, require more
 181 complex processing. We first summarize rules supported by
 182 our system (illustrated in Figure 3), and then we present some
 183 additional implementation issues including a more detailed de-
 184 scription for non-trivial rules.

185 **Line Detection** We estimate a path’s deviation from straight-
 186 ness by measuring the ratio between its length and the distance
 187 between its endpoints, as in QuickDraw [10].

188 **Arc Detection** We sample the input path and perform a least-
 189 squares circle fit on the samples to obtain center and radius pa-
 190 rameter values. To determine the angular span value, we project
 191 the samples onto the circle fit. The arc is then sampled again
 192 and we evaluate the discrete Fréchet distance [18] between the
 193 arc samples and the samples of the input path. When the span is
 194 close to 2π or the path is closed, we replace it with a full circle.

195 **Endpoint Snapping** We look at the distance between each of
 196 the path endpoints and resolved endpoints. Additionally, we
 197 also try snapping to inner parts of the resolved paths. Special-
 198 ized tests based on the properties of line segments and circular
 199 arcs lower the computational complexity of this operation. Note
 200 that we do not join the two end-to-end-snapped paths. This can
 201 cause unpleasant artifacts where they meet, but the effect of a
 202 join can be mimicked by using round end caps on the strokes.

203 **End Tangent Alignment** If the path endpoint is snapped, we
 204 measure the angle between its tangent and the tangent of the
 205 point it is attached to.

206 **Line Parallelism and Perpendicularity** We compare the an-
 207 gle between two line segment paths with the angle needed to
 208 satisfy the parallelism or perpendicularity constraint. Addition-
 209 ally, we also take the distance between the line segments into
 210 account to slightly increase the priority of nearby paths. To
 211 evaluate these properties on the input non-rectified paths, we
 212 use their line segments approximations, i.e., line segments con-
 213 necting their two endpoints.

214 **Line Length Equality** We evaluate the ratio of length of both
 215 tested line segments. As in previous case, we incorporate their
 216 mutual distance in the final likelihood computation.

217 **Arc and Circle Center Snapping** Similar to endpoint snapping,
 218 we evaluate the distance between the current arc center and po-
 219 tential ones, in this case endpoints of other paths, other centers,
 220 centers of rotations, and centers of regular polygons composed
 221 from series of line segments. However, as arcs with small angu-
 222 lar span are noticeably harder to draw without a guide (see Fig-
 223 ure 6a), the center of the initial arc fit might be located too far
 224 apart from the desired center point (Figure 6b) and therefore us-
 225 ing fixed distance, when looking for potential center-snapping
 226 points, might not be sufficient. To address this issue, we adap-
 227 tively change this distance to $\max(D, 2r(1 - \theta/2\pi))$, where θ
 228 is the span of the tested arc, r is its radius and D is the stan-
 229 dard search distance radius ($D = 30$ view-space pixels in our
 implementation).

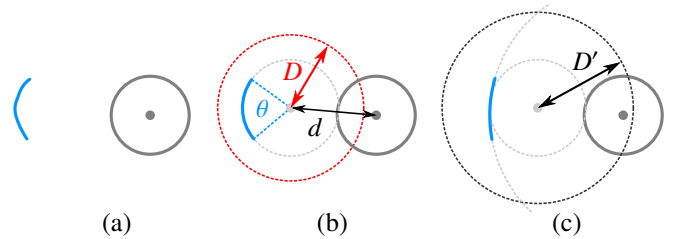


Figure 6: Adaptive arc/circle center-point-snap search distance refinement. Arc segments with small angular span are often drawn very imprecisely (a). When the engine fits an exact arc into such data, its center is often too far from the desired center point, as the distance d between them is bigger than the limit D under which the prospective center point positions are looked for (b). Adaptive expansion of the search radius D' increases the likelihood that even the imprecise input will give the user the expected (precise) output.

230
 231 **Path Identity** To detect that two paths have similar shapes, we
 232 align them and compute their discrete Fréchet distance. More
 233 details are given in Section 3.4.

234 **Transformation Adjustment** For a tested path x and resolved
 235 reference path y of the “same shape” (determined by successful
 236 application of the path-identity rule) we perform a variety of
 237 modifications to the transformation to create symmetries, align
 238 paths, and equalize spacing. More details are given in Sec-
 239 tion 3.5.

240 **Path Offset** Offset paths generalize line parallelism. To detect
 241 them, we go along the tested path and measure its distance to
 242 the reference path. More details are given in Section 3.6.

243 3.2. View-Space Distances

244 Testing paths for different geometric properties ultimately
 245 requires measuring lengths and distances. While many path at-
 246 tributes can be compared using relative values, absolute values
 247 are still necessary, e.g., for snapping endpoints. Using abso-
 248 lute values, however, leads to unexpected behavior when the
 249 canvas is zoomed in and out. To eliminate this problem, we
 250 compute all distances in view-space pixels, making all distance
 251 tests magnification-independent.

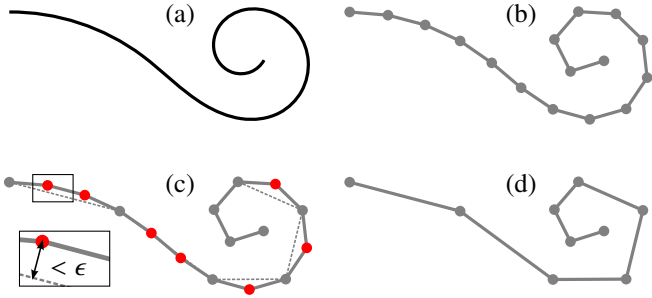


Figure 7: Path sample simplification. The original Bézier path (a) is equidistantly sampled, giving a polyline (b). The Ramer–Douglas–Peucker algorithm then recursively simplifies the polyline by omitting points closer than ϵ (c) to the current approximation, finally constructing simplified polyline (d).

252 3.3. Path Sampling

253 Working with cubic Bézier curves analytically is inconvenient and difficult. Many practical tasks, such as finding a path’s
 254 length or the minimal distance between two paths, can only be
 255 solved using numerical approaches. Therefore, we perform all
 256 operations on sampled paths. Since the resolved paths do not
 257 change, we can precompute and store the samples for resolved
 258 paths, and sample only new paths. Furthermore, to reduce the
 259 memory requirement and computational complexity of different
 260 path comparisons, we simplify the sampling using the *Ramer–*
 261 *Douglas–Peucker* algorithm [19, 20]. For a polyline p , this
 262 finds a reduced version p' with fewer points within given tol-
 263 erance ϵ , i.e., all points of p' lie within the distance ϵ of the
 264 original path (see Figure 7). Our implementation uses $\epsilon = 4$
 265 view-space pixels at the time the path was drawn.
 266

267 3.4. Path Matching

268 A key part of our contribution involves resolving higher-
 269 level geometric relations like path rotational and reflection sym-
 270 metry. To identify these relations, we must first classify paths
 271 that are the “same shape”—paths that are different instances of
 272 the same “template”.

To evaluate the similarity between two sampled paths p_a
 and p_b , we employ a discrete variant of *Fréchet distance* [18],
 a well-established similarity measure. Formally, it is defined as
 follows: Let (M, d) be a metric space and let the path be defined
 as a continuous mapping $f : [a, b] \rightarrow M$, where $a, b \in \mathbb{R}, a \leq b$.
 Given two paths $f : [a, b] \rightarrow M$ and $g : [a', b'] \rightarrow M$, their
 Fréchet distance δ_F is defined as

$$\delta_F(f, g) = \inf_{\alpha, \beta} \max_{t \in [0, 1]} d(f(\alpha(t)), g(\beta(t))), \quad (2)$$

273 where α (resp. β) is an arbitrary continuous non-decreasing
 274 function from $[0, 1]$ onto $[a, b]$ (resp. $[a', b']$). Intuitively, it is
 275 usually described using a leash metaphor: a man walks from
 276 the beginning to the end of one path while his dog on a leash
 277 walks from the beginning to the end of the other. They can
 278 vary their speeds but they cannot walk backwards. The Fréchet
 279 distance is the length of the shortest leash that can allow them
 280 to successfully traverse the paths.

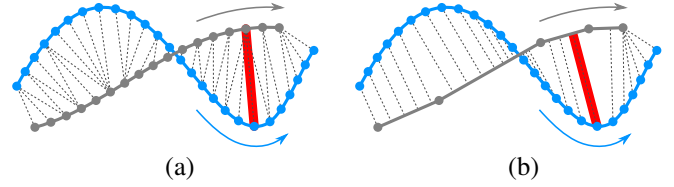


Figure 8: Discrete Fréchet distance. The minimum length of the line connecting ordered sets of point samples (a). Since we store the resolved paths in the simplified form, we compute the Fréchet distance between an ordered set of points and an ordered set of line segments (b) rather than between two point sets.

281 As outlined by Eiter and Mannila, this can be computed for
 282 two point sets using a dynamic programming approach. The
 283 extension to point and line-segment sets (Figure 8b) is then
 284 straightforward. However, the measure takes into account the
 285 absolute positions of the sample points, while we are inter-
 286 ested in relative difference. Therefore, we have to adjust the
 287 alignment of the two tested paths. We then compute the dis-
 288 crete Fréchet distance between the aligned paths, divided by the
 289 length of the new path to obtain the relative similarity measure.

290 An affine similarity transform is a composition of a rotation,
 291 a uniform scale, and a translation. To align the paths, we find
 292 the affine similarity matrix that transforms the reference path to
 293 match the new path as closely as possible.

294 Assume the rotation angle is θ , the scale is s , and the trans-
 295 lation is (tx, ty) . Define $scos = s * \cos \theta$ and $ssin = s * \sin \theta$.
 296 The matrix is then

$$\begin{bmatrix} scos & -ssin & 0 \\ ssin & scos & 0 \\ tx & ty & 1 \end{bmatrix} \quad (3)$$

297 We compute the affine similarity transformation matrix M
 298 as follows. We first create two equal-length lists of points, each
 299 consisting of N equally-spaced samples from the reference and
 300 new paths. If $\{P_i\}$ are the points from the reference path and $\{Q_i\}$
 301 the points from the new path, we find the M that minimizes the
 302 sum of the squared distances

$$E = \sum_{i=1}^N \|P_i * M - Q_i\|^2 \quad (4)$$

303 This is a quadratic function of $scos$, $ssin$, tx , and ty and can
 304 be solved as a least-squares problem over these four variables.

305 Before computing the Fréchet distance, we multiply the ref-
 306 erence path samples by M . If the Fréchet distance indicates that
 307 the paths are sufficiently similar, we create a suggestion consist-
 308 ing of the reference path transformed by this same M .

309 A path that is a transformed copy of another path is perma-
 310 nently annotated as such, thereby allowing us to optimize path
 311 matching by only testing against a single instance of the path.
 312 For later processing, we also annotate the path with the trans-
 313 formation matrix.

314 If the drawing already contains multiple instances of a path,
 315 we consider it more likely that the user intended a new path

316 to match. We therefore boost its score s by replacing it with
 317 $1 - (1 - s)^{ln i}$ where i is the number of existing instances.

318 Because the new path might be a reflected and/or reversed
 319 version of the reference path, we perform four tests between
 320 them to determine the correct match.

321 3.5. Transformation Adjustment

322 If the test path is a transformed version of a reference path,
 323 there are various tests we perform to adjust the transformation
 324 matrix to make the result more pleasing. We first begin by separating the matrix in Equation 3 into separate rotation, scale, and translation components as follows:

$$\begin{aligned} \text{rotation} &= \text{atan2}(s\sin, s\cos) \\ \text{scale} &= \sqrt{s\cos^2 + s\sin^2} \\ \text{translation} &= (tx, ty) \end{aligned} \quad (5)$$

327 The transformation can be adjusted in various ways, often
 328 generating multiple suggestions. Although we optimized path
 329 matching to only compare against one instance of a path that
 330 has multiple copies in the drawing, we test the transformation
 331 relative to each copy; see Figure 9a.

332 **Rotation Snapping** If the rotation component is close to an
 333 angle that is an integral divisor of 2π , it is snapped to being that
 334 angle (e.g., to 45 degrees; see Figure 10b4).

335 **Scale Snapping** If the scale component is close to an integer
 336 or to 0.5, it is snapped to being that exact scale.

337 **Translation Snapping** Translation snapping takes several forms:

- 338 • If the transformation contains a rotation component, we
 339 find the rotation center and compare it to existing points
 340 in the drawing. If it is sufficiently close we adjust the
 341 translation to place the center of rotation at that point.
- 342 • If the test path is a reflected version of the resolved path,
 343 we first compute the axis of reflection and reflect the resolved
 344 path across this axis. If the test path is sufficiently
 345 close to this reflected path, we adjust the translation to
 346 move it to that position.
- 347 • In other cases, we snap the x and y components of the
 348 translation to zero.

349 **Step Transform Snapping** Step transform snapping allows the
 350 user to create multiple, equally transformed copies of a path
 351 (see Figure 10b3). When we snap a path to an instance of a
 352 path, we store the relative transformation to that instance as the
 353 *step transform*. The step transform is the relative transform of
 354 the most highly-scoring suggestion. In Figure 9b, the exist-
 355 ing drawing contains three resolved paths that are all the same
 356 shape. R was drawn first, and is the reference path. C is the
 357 first copy, and its step transform is the transformation from R
 358 to C . D is the second copy, and it was horizontally snapped
 359 to C . Because the transformation from C scored more highly

360 (containing a snap) than the transformation from R , the step
 361 transform for D is the relative transform from C to D .

362 Step transform snapping compares the transformation from a
 363 path instance to the step transform for that instance. If the two
 364 transformations are similar, then a step-snapping suggestion is
 365 generated. In Figure 9c, the newly drawn path T is compared
 366 to all three existing instances R , C and D . The transformation
 367 M_{DT} from D to T is similar to the step transform of D . This
 368 generates a step-snapping suggestion to place T in the position
 369 that exactly matches the step transform; see Figure 9d.

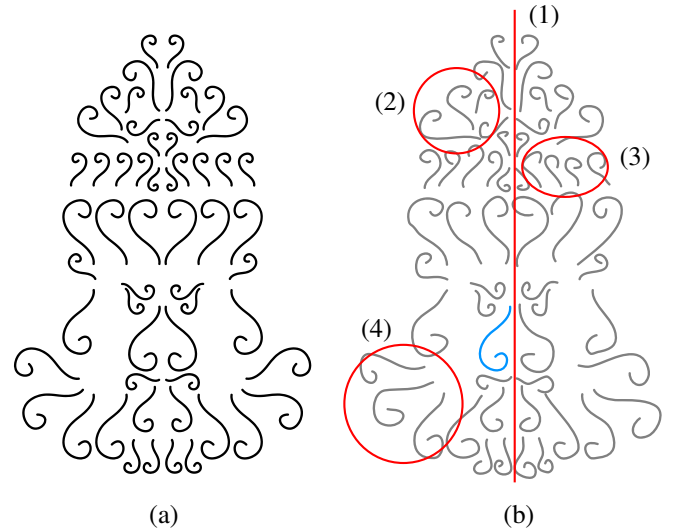


Figure 10: Practical application of transformation adjustment of the imprecise input (b) to obtain highly symmetrical output (a). We apply reflection axis (1), step transform (2,3) and rotation (2,4) snapping. Also note that the whole drawing is composed of strokes of the same shape.

370 Although this example only includes translation in the step trans-
 371 form, they are fully general, and can include rotation, scale, and
 372 reflection (see Figure 10b2).

373 **Reflection Axis Snapping** Users often want to reflect multiple
 374 paths against the same axis of reflection (for example, see the
 375 bear in Figure 1), or want to reflect a path across an existing
 376 line segment. To accommodate this, we collect all existing axes
 377 of reflection and line segments. If the new path is reflected,
 378 we compare its axis of reflection to these potential axes, and if
 379 it is close, we generate a suggestion to reflect across this axis
 380 (see Figure 10b1). Further, we strengthen the likelihood for an
 381 axis that has already been used multiple times by replacing the
 382 score s with $1 - (1 - s)^{ln i}$ where i is the number of times that
 383 axis has been used.

384 3.6. Offset Path Detection

385 Offset paths extend the concept of parallelism from line seg-
 386 ments to paths. To detect them, we construct a normal line from
 387 each sample of the new path. If the line hits an existing refer-
 388 ence path, we measure the distance between the sample point
 389 and the closest point on the reference. Note that we do not use

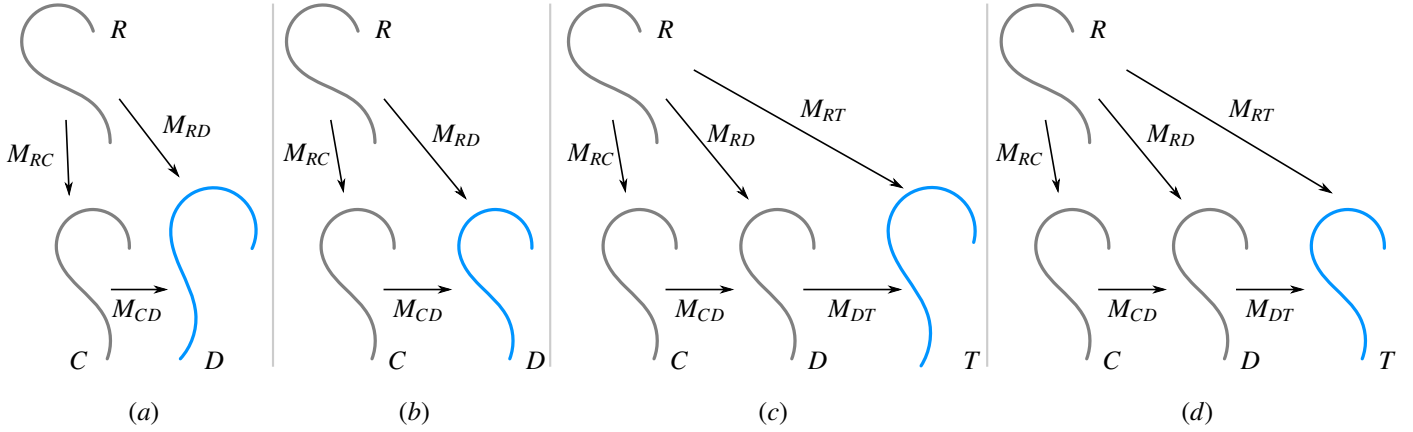


Figure 9: Transformation adjustment and transformation step snapping. The reference path R already has a copy C in the drawing, with M_{RC} being the transformation from R to C . D is the test path with M_{RD} being the transformation from R to D . Transformation adjustment considers both M_{RD} and the derived relative matrix M_{CD} that transforms C to D (a). The step transform for D is then M_{CD} , the relative transform from C (b). The relative transform for T relative to D is similar to the step transform for D (c). Applying M_{CD} to D generates a well-spaced suggestion (d).

390 the distance between the sample point and the line-path inter-
 391 section, since this would require the user to draw the approxi-
 392 mate offset path very precisely. We store the measured distance
 393 along with its sign, i.e., on which side of the new path the hit
 394 occurred. We then sort all the hit information according to the
 395 distance, creating a cumulative distribution function, and pick
 396 two values corresponding to $(50 \pm n)$ -th percentiles (n being 25
 397 in our implementation). By comparing the sign and distance
 398 values of these samples, we calculate the likelihood of the new
 399 path being an offset path of the reference path (see Figure 11).
 400 If the likelihood is high, we replace the new path with an offset
 401 version of the reference.

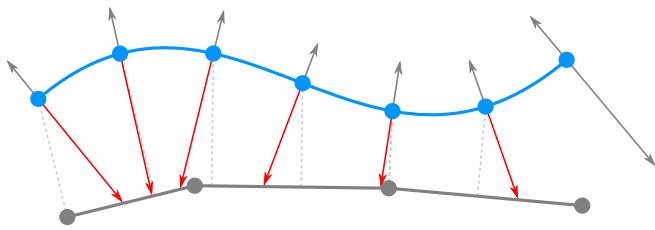


Figure 11: Offset path detection. A line is constructed from each point on the sampled path (blue circles) in the normal direction. If an existing reference path is hit (red rays), the minimal distance from the sample to the reference path is calculated (dashed lines) and used in offset-path-likelihood computation (see 3.6).

402 4. Multi-Segment Stroke Processing

403 The single stroke processing approach gives the user the
 404 opportunity to immediately see the results of the input being
 405 beautified. However, in certain cases, like drawing simple tri-
 406 angles or squares, this workflow can be tedious and decrease the
 407 overall fluency of the beautification pipeline. To this end, we
 408 introduce an additional step into our scheme that lets the eval-
 409 uation engine process strokes with multiple segments. These

410 segments are defined as parts of the unprocessed user input,
 411 split by corner features. Once divided, the evaluation engine
 412 can process the simple segments using the geometric rules in-
 413 troduced in Section 3.1.

414 4.1. Corner Detection

415 When the raw freehand input stroke is drawn by the user, it
 416 is converted to a sequence of cubic Bézier curves and passed to
 417 the beautification pipeline. The first step is to test it for the pres-
 418 ence of corner points. Because the initial curve fitting is done by
 419 the host application (e.g., Adobe Illustrator), we cannot simply
 420 rely on the assumption that corners can only occur at the junc-
 421 tion of two Bézier curves. For example, in Figure 12a, the ap-
 422 parent corner in the lower right is actually a small-radius curve.
 423 We initially sample the curves with a small step size (2 view-
 424 space pixels) and calculate the tangent vector at each sample
 425 point. Using a sliding window of three successive samples, we
 426 calculate the angular turn value at every sample position ex-
 427 cept the first and last. Local maxima in this turn sequence provide
 428 the places to break the original input sequence into segments.
 429 To handle outliers like the unwanted “hooks” at the ends, we
 430 discard segments whose length is small compared to the rest
 431 of the segments (less than 15% of the length-wise closest other
 432 segment).

433 4.2. Segment Processing

434 The segments of the complex user input can then be pro-
 435 cessed one at the time using the same approach used for the sim-
 436 ple input described in Section 3. There are, however, important
 437 issues to address. Most notably, processing multi-segment in-
 438 put involves automatic selection of intermediate outputs, which
 439 would otherwise be done by the user. As the number of po-
 440 tential outputs rises exponentially, we cannot explore the whole
 441 search space. Therefore, we perform two reduction steps to
 442 make the evaluation of complex inputs computationally feasible
 443 within real-time-to-interactive response time. First, we limit the
 444 number of unique suggestions for each segment to 3 (whereas

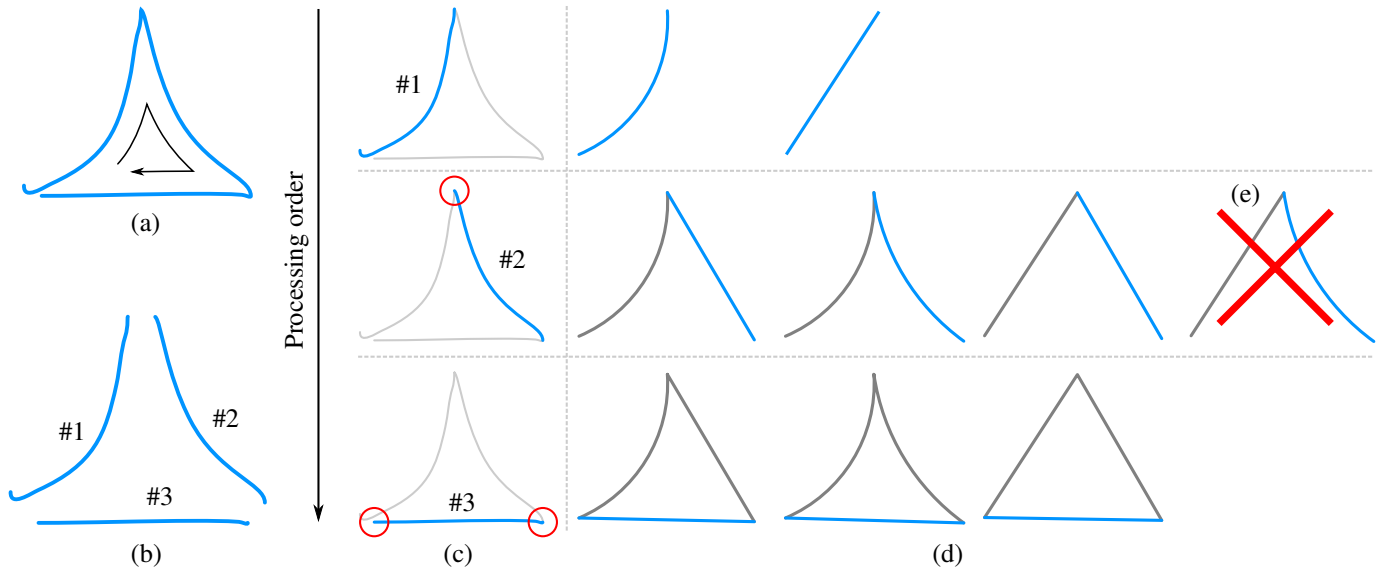


Figure 12: Multi-segment stroke processing pipeline. When a complex stroke is drawn (a), it is tested for the presence of corner points. If no corner points are found, the processing continues as described in Section 3. If one or more corner points are detected (see Section 4.1 for more details), the original stroke is split and broken into segments (b). The segments are then processed sequentially. After each individual segment is added (c, from top to bottom), suggestions are generated (d) using previous segments as well as old strokes. In particular, beginning with the second segment, the beginning endpoint is constrained to match the final endpoint of the previous segment (c, red circles, see Section 4.3). After generating suggestions for a segment (d, from top to bottom), an optional set reduction can be done (e) to keep the evaluation sufficiently fast (see Section 4.2).

445 the single-segment input can produce up to 10 suggestions).
 446 This might seem to be a very severe restriction, but the split seg-
 447 ments are typically simple paths with very little ambiguity. Sec-
 448 ond, we process the individual segments in a breadth-first man-
 449 ner that lets us execute another reduction once all the parallel
 450 states reach the same depth (i.e., they all have the same number
 451 of processed segments; see individual rows in Figure 12d). For
 452 this step, we assign each intermediate state a value calculated
 453 as the arithmetic mean of the scores of the processed segments.
 454 Then, only N_{IS} intermediate states are kept and evaluated fur-
 455 ther while the rest are discarded (Figure 12e). The performance
 456 of multi-segment input processing is determined by the number
 457 of segments K and the intermediate stack size N_{IS} , with
 458 $N_{IS} = 1$ being performance-wise equal to sequential process-
 459 ing of individual segments. In our implementation, $N_{IS} = 10$
 460 and strokes constituted of up to 10 segments can be processed
 461 without noticeable lagging.

462 4.3. Internal Segment Restrictions

463 As the individual segments are pieces of one original input
 464 curve, we must ensure that the beautified segments are consec-
 465 utively joined. Thus, we constrain the position of the first end-
 466 point of each segment after the first (rows 2,3 in Figure 12c).
 467 Additionally, if the input stroke is closed, we also constrain the
 468 last segment's final endpoint (row 3 in Figure 12c). As a side
 469 effect, this also helps to decrease the ambiguity.

470 4.4. Segment Joining And Further Behavior

471 Once all the segments have been processed, we create the
 472 final output stroke by joining them together. This way, the com-
 473 bined beautified input stroke can be used by rules such as curve

474 identity. Internally, the beautification engine keeps also tracks
 475 the individual segments so that they behave as if they were
 476 drawn one after each other. This lets the geometric rules show
 477 the expected behavior, e.g., the corners of a complex stroke can
 478 be used as snapping points.

479 5. Implementation Details

480 While using an existing API requires us to conform to its
 481 design rules, it also eliminates the need to handle many tasks
 482 unrelated to the research project, such as tracking the input de-
 483 vice, fitting paths to the samples, and managing the undo/redo
 484 stack. It also benefits the users, as they are not forced to learn
 485 yet another user interface, and can instead take advantage of
 486 built-in tools of the existing program. Therefore, we decided to
 487 integrate our system into Adobe Illustrator as a plugin using its
 488 C++ SDK.

489 As described previously, our method is based on evaluat-
 490 ing different geometric rules on a new path using the previously
 491 drawn and resolved paths. Thus, we need to be able to detect
 492 when a new path is created or an old one is modified or deleted.
 493 To this end, we serialize all the path data and store a copy in the
 494 document. Illustrator activates our system whenever the user
 495 modifies the document. We deserialize the data and compare
 496 the paths to the actual paths in the document to detect changes.
 497 If we find a new path, we process the new path and update the
 498 serialized data. Similarly, when a path is modified, it is treated
 499 as new one and reprocessed. Deleting paths does not affect the
 500 remaining ones. To support undo and redo, we store the se-
 501 rialized data into a part of document that is managed by the
 502 undo/redo system.

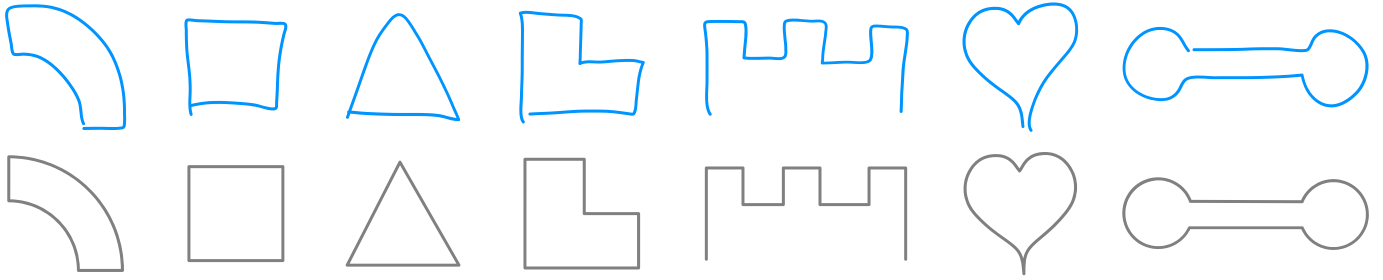


Figure 13: Examples of multi-segment stroke processing. The input strokes (blue) are broken into individual segment that are sequentially processed using the single-segment evaluation engine (Section 3) and merged after the processing is finished (see Section 4 for details).

503 The presentation of the suggestions is deliberately kept as
 504 simple as possible and only one suggestion is shown at the time.
 505 The user switches among the suggestions using an additional Il-
 506 lustrator tool panel. The last suggestion in the list is always the
 507 original input path and is thus easily accessible. Currently, the
 508 list of inferred constraints is shown in textual form in the order
 509 in which they were traversed in the search space tree (see Fig-
 510 ure 18c). The user selects the current suggestion by drawing a
 511 new path or changing the selection. To provide additional as-
 512 sistance for the user, we also present a simple visualization of
 513 the applied rules together with rectified path. This visual an-
 514 notation provides immediate feedback about the imposed con-
 515 straints and relations of the user input (see Figure 15).

516 To further exploit the built-in tools, we support the “Trans-
 517 form Again” feature for rotational symmetry. If the resolved
 518 path is a rotated copy of an existing path, it is noted as such
 519 so that a new, properly-rotated copy will be created if the user
 520 invokes the “Transform Again” command. The user only needs
 521 to draw two rotated instances of a path and then can create ad-
 522 ditional properly-rotated paths without drawing them (see Fig-
 523 ure 18d). Recall that the rotation angle is adjusted to the nearest
 524 integer quotient of 2π , so additional paths can form full n -fold
 525 rotational symmetry.

526 The constraints imposed by ShipShape can easily be avoided
 527 for certain paths by placing them in layers that are not being
 528 rectified. In our implementation, ShipShaperuns only on the
 529 default layer.

530 6. Results

531 To evaluate the effectiveness of our method, we conducted
 532 a preliminary study. We created a plugin for Adobe Illustrat-
 533 or that was installed on a PC with a 23in LCD monitor and a
 534 consumer computer mouse as the input device. Six people par-
 535 ticipated in this study. All of them worked with Illustrator on
 536 a daily to weekly basis, but in all cases, their primary work-
 537 related tool was a CAD program. First, the users were given a
 538 brief introduction and demonstration of our system’s concept,
 539 capabilities and limitations, with a few practical examples. The
 540 participants could adjust Illustrator settings and the mouse sen-
 541 sitivity according to their needs, and then spent 1 to 3 minutes
 542 in free drawing, to get briefly accustomed to the system and the
 543 workflow.

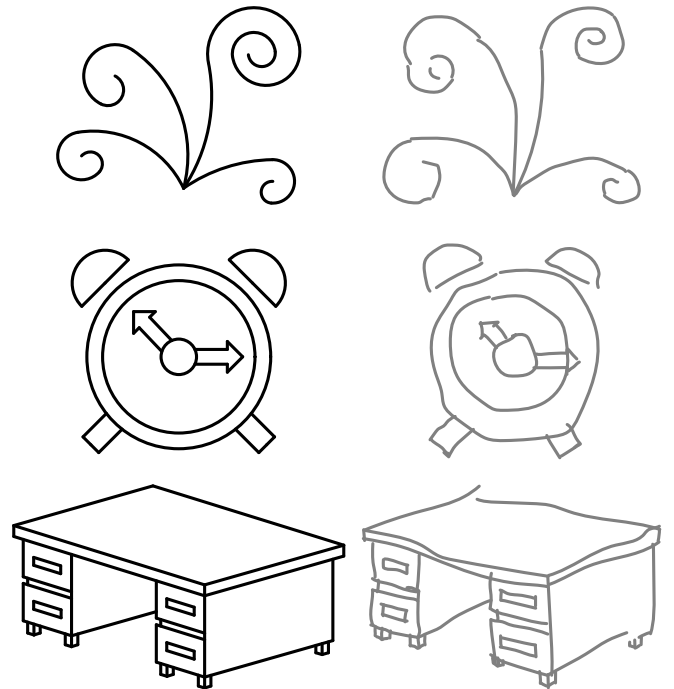


Figure 14: Evaluation study drawings. The users were asked to recreate these drawings using our ShipShape prototype: Task drawing (left, black), representative raw input (right, gray).

544 The users were then shown three simple illustrations (see Fig-
 545 ure 14) and presented with the task of drawing each of them
 546 anew, using both native Illustrator tools and our prototype, while
 547 we measured their drawing times. First, the participants were
 548 asked to recreate the figures using any suitable tools and ap-
 549 proaches, i.e., they could use all the available tools and modes,
 550 such as copying or reflecting. Rather than creating the exact
 551 copies of the reference drawings, we directed them to focus on
 552 preserving the geometric relations. Interestingly, despite the
 553 users’ relatively equal level of experience, they often took very
 554 dissimilar ways to recreate the task’s drawing.

555 In the second part of the test, the participants were only
 556 allowed to use the pencil tool with the ShipShape prototype
 557 turned on. The only additional allowed operation was undo.
 558 Similarly to the first part of each drawing, the users took a dif-
 559 ferent approaches to complete the goal, however, with a sin-

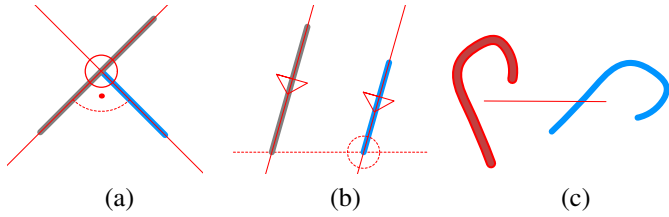


Figure 15: Visual annotation hints. Overlaid visual annotations show which rules have been applied, e.g., line perpendicularity and endpoint snapping (a), line parallelism and single coordinate snapping (b) or path identity (c).

560 gle exception, they were all able to finish all three designated
 561 drawings from Figure 14. The initial measurements (Figure 16)
 562 suggest that drawing beautification is more suited for simpler
 563 drawings and tasks. For example, copying a large part of the
 564 bottom-left drawing in Figure 14 was always faster than redrawing
 565 it.

566 The main interest of this study was to identify the weak
 567 points and bottlenecks of our approach and to test how success-
 568 ful our prototype was in generating correct suggestions. The
 569 overall feedback from the participants was positive. They found
 570 the tool useful and easy to use. Most of the participants, how-
 571 ever, considered the limitation of using a single tool only too
 572 restrictive, and suggested incorporating parts of our approach
 573 (smart snapping, automatic tangent adjustments, etc.) into the
 574 relevant built-in tools. All the participants liked the idea of vi-
 575 sual annotations (Figure 15) and found it helpful. Several users
 576 did not like the way the alternative suggestions were presented
 577 and explored (see the small gray box in canvas in Figure 18)
 578 and preferred to undo and redraw the particular strokes.

579 Additional results are shown in Figure 17. Note that an im-
 580 portant part of the drawing workflow was relying on Illustrat-
 581 or’s built-in support for curve smoothing when creating origi-
 582 nal paths—those that are not copies of other paths. These are
 583 shown in blue in Figure 17, and they function as “template”
 584 paths for the beautification. Other strokes drawn afterwards
 585 can be much more imprecise (see Figure 1 and Figure 17c–g).

586 7. Limitations and Future Work

587 A common problem of drawing beautification techniques is
 588 the quick growth of the number of possible suggestions as the
 589 drawing becomes more complex and many satisfiable geomet-
 590 ric constraints emerge. Our approach addresses this by combin-
 591 ing best-first search with a limited suggestion set size, but
 592 additional heuristic-based pruning of the search space, possibly
 593 based on empirical measurements, could improve the sugges-
 594 tion set.

595 Currently, when the user changes an already-resolved path,
 596 it is treated as a new one. In some cases, however, it would be
 597 beneficial to not only reprocess the modified path but also all
 598 other paths being in relationship with it, for example changing
 599 any reflected or rotated versions of the path.

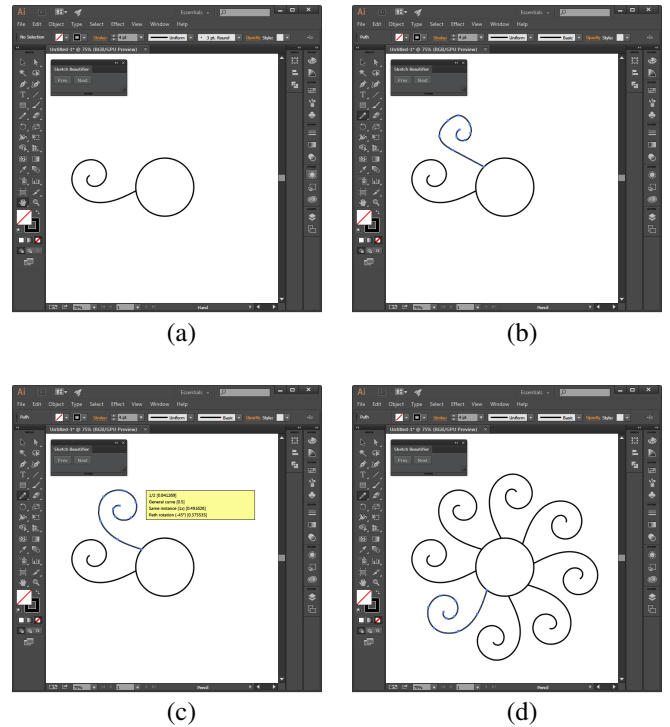


Figure 18: Exploiting the “Transform Again” feature. Illustrator allows the user to repeat the last transformation. When a new path is added (b) to the canvas (a), it is processed and output suggestions are generated. If the user chooses a suggestion that is a rotation (c) we enable the “Transform Again” feature. The user can then easily complete the 8-fold rotational symmetry drawing (d). See Section 5

600 8. Conclusion

601 In this paper, we presented an efficient method for beautifi-
 602 cation of freehand sketches. Since the user input is often impre-
 603 cise and thus ambiguous, multiple output suggestions must be
 604 generated. To this end, we formulated this problem as search
 605 in a rooted tree graph where nodes contain transformed input
 606 stroke, edges represent applications of geometric rules and suit-
 607 able suggestions correspond to different paths from root node
 608 to some leaf nodes. To avoid the computational complexity of
 609 traversal through the whole graph, we utilized a best-first search
 610 approach where the order of visiting tree nodes is directed by
 611 the likelihood of application of the particular geometric rules.

612 On top of this framework, we developed a system of self-
 613 contained rules representing different geometric transformations,
 614 which can be easily extended. We implemented various rules
 615 that can work not only with simple primitives like line segments
 616 and circular arcs, but also with general Bézier curves, for which
 617 we showed how to detect previously unsupported relations such
 618 as curve identity or rotational and reflection symmetry.

619 We demonstrated the usability and potential of our method
 620 on various complex drawings. Thanks to the ability to process
 621 general curves, our system extends the range of applicability
 622 of freehand sketching, which was limited previously to draw-
 623 ings in specialized, highly-structured applications like forms or
 624 technical diagrams. We believe that this advantage will become

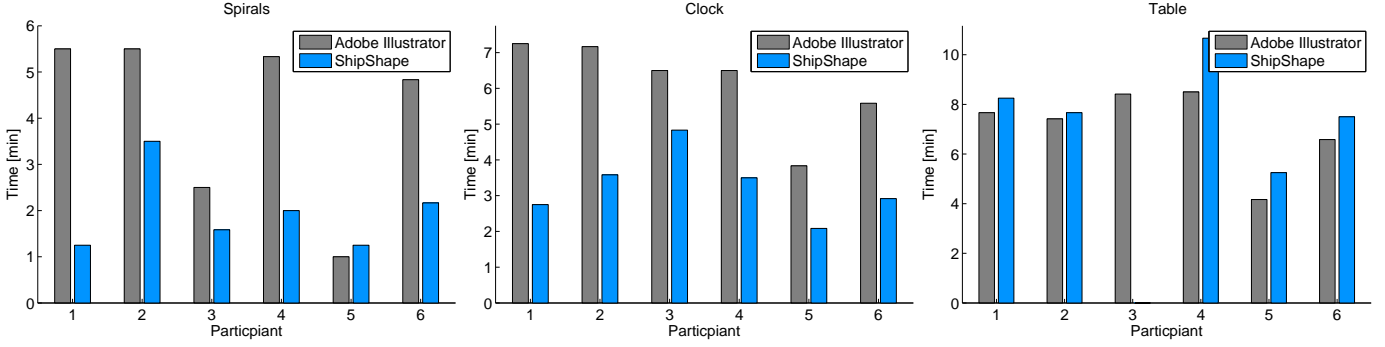


Figure 16: Comparison of drawing performance. The participants were asked to recreate the drawings from Figure 14 using either native tools of Adobe Illustrator (red) or ShipShape prototype (blue line). For simpler drawing, such as the spiral or the clock, ShipShape typically outperformed Illustrator. However, with more complex drawings (table), the utilization of different tools is faster.

625 even more apparent with the widespread adoption of touch-
 626 centric devices, which rely much less on classical beautification
 627 techniques that are based upon menu commands and multiple
 628 tools.

629 9. Acknowledgements

630 We would like to thank all the anonymous reviewers for
 631 their constructive comments. This research began as an intern-
 632 ship by Jakub Fišer at Adobe Research, and was partly sup-
 633 ported by the Technology Agency of the Czech Republic under
 634 the research program TE01020415 (V3C – Visual Computing
 635 Competence Center) and partially by the Grant Agency of the
 636 Czech Technical University in Prague, grant No. SGS13/214/-
 637 OHK3/3T/13 (Research of Progressive Computer Graphics Meth-
 638 ods).

639 Appendix A. Rules Evaluation

640 The rules are evaluated using a piecewise-linear ramp func-
 641 tions, both continuous and discontinuous. These functions trans-
 642 form the input values, such as angular differences or view-space
 643 distances, to likelihood values from the interval $[0, 1]$ used to di-
 644 rect the tree expansion and final suggestion sorting described in
 645 the paper. For each rule listed in section 3.1 in the main paper,
 646 we show the exact scoring function we used in our implemen-
 647 tation.

648 Appendix A.1. Line Detection

649 As in QuickDraw [10], we calculate the deviation from straight-
 650 ness $D = |1 - |l_c|/|l_l||$, where $|l_c|$ is length of sampled Bézier
 651 curve and $|l_l|$ length of line segment between its endpoints. If D
 652 is lower than the threshold 0.05, we set the likelihood \mathcal{L}_{LD} of
 653 the curve being a line segment to $1 - D$.

654 Appendix A.2. Arc Detection

655 The arc is described by three parameters – center, radius
 656 and angular span. We initially sample the input path and obtain
 657 the circle fit center location and radius value using least-squares
 658 approach. We then project the samples onto the optimal circle,

659 using the circle center as the center of the projection, to deter-
 660 mine the span value. Having these three values, we construct
 661 the arc suggestion and compute its similarity with the input us-
 662 ing discrete Fréchet distance between the original samples and
 663 the suggested arc’s samples. This distance is then used to cal-
 664 culate the final output likelihood \mathcal{L}_{AD} (Figure A.19). If the de-
 665 tected span is higher than $2\pi - \pi/13$ or the input path is closed,
 the output span is set to 2π to suggest full circle output.

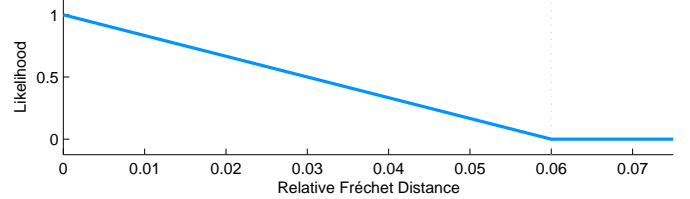


Figure A.19: Relative discrete Fréchet distance evaluation in *Arc Detection* rule.

666 Appendix A.3. Endpoint Snapping

667 We measure the distances between the endpoint and the
 668 points of interest (other endpoints, arc centers, etc.) in view-
 669 space pixels and transform them to final likelihoods \mathcal{L}_{ES} (Fig-
 670 ure A.20). As the users can end strokes relatively precisely even
 671 with devices such as mouse or touchpad, there is no tolerance
 672 zone in the scoring function.
 673

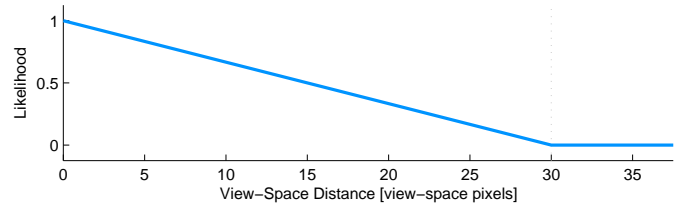
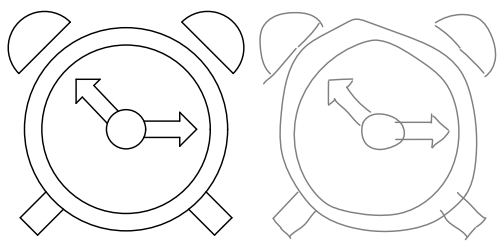


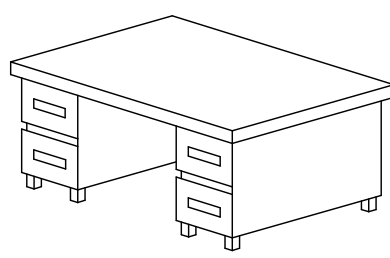
Figure A.20: View-space distance evaluation in *Endpoint Snapping* rule.

674 Appendix A.4. End Tangent Alignment

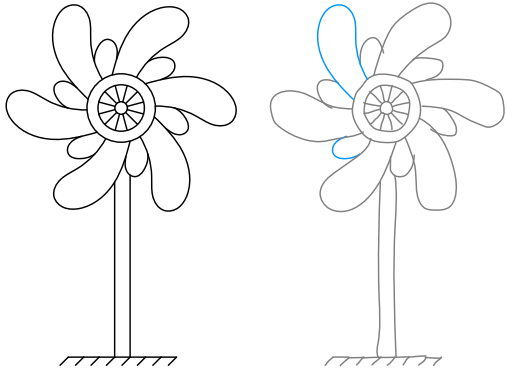
675 The angular difference between the curve endpoint and the
 676 endpoint it is connected to is directly transformed to final like-
 677 lihood \mathcal{L}_{ETA} (Figure A.21).



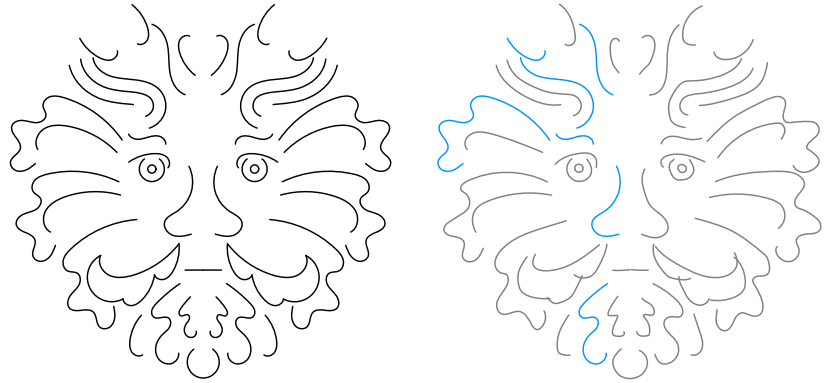
(a)



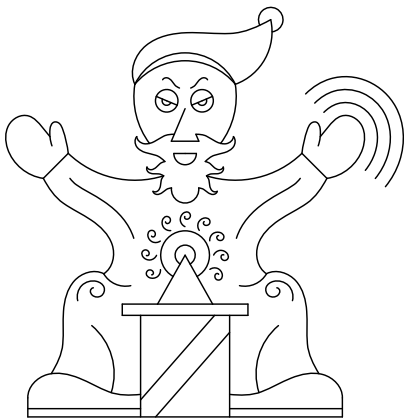
(b)



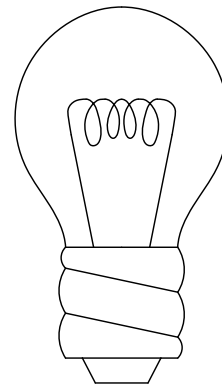
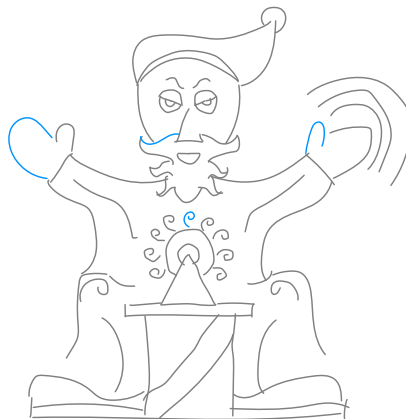
(c)



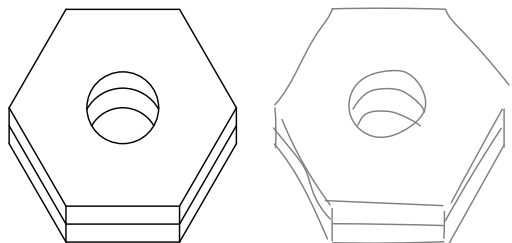
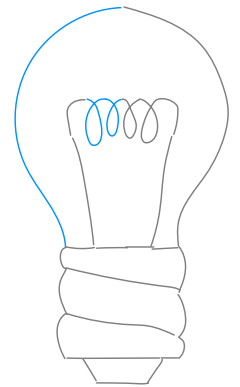
(d)



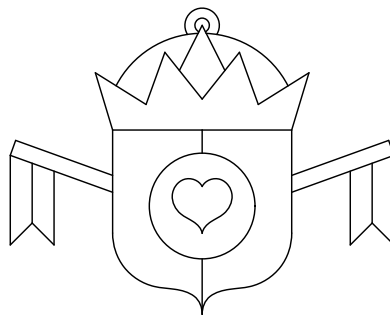
(e)



(f)



(g)



(h)

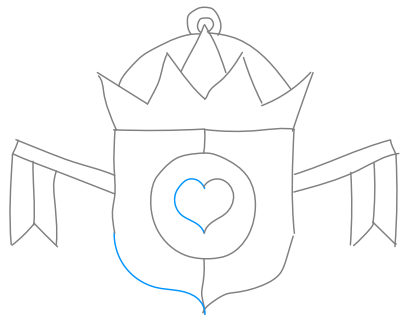


Figure 17: Various results obtained using our method. The side-by-side pairs show the beautified output (black) and the original input strokes (gray). Note that we do not perform any curve smoothing, beyond what is provided by Illustrator. Therefore, when dealing with general curves, the first “template” strokes (blue) have to be drawn more precisely or be smoothed using built-in Illustrator capabilities.

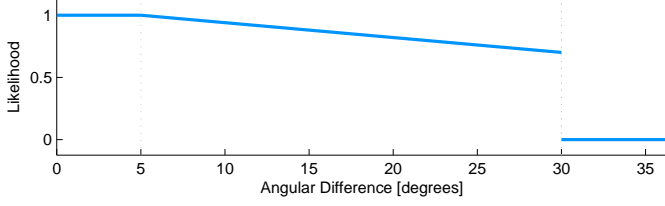


Figure A.21: Angular difference evaluation in *End Tangent Alignment* rule.

678 Appendix A.5. Line Parallelism and Perpendicularity

679 We measure the angular difference between the direction
680 vectors of two line segments to obtain the likelihood \mathcal{L}_{dff} (Fig-
681 ure A.22 top). To increase the final likelihood of nearby line
682 segments, we also score the view-space distance between tested
683 line segments – \mathcal{L}_{dst} (Figure A.22 bottom). The output sugges-
684 tion with likelihood $\mathcal{L}_{LP} = \mathcal{L}_{dff}\mathcal{L}_{dst}$ is produced, if $\mathcal{L}_{LP} > 0.7$.

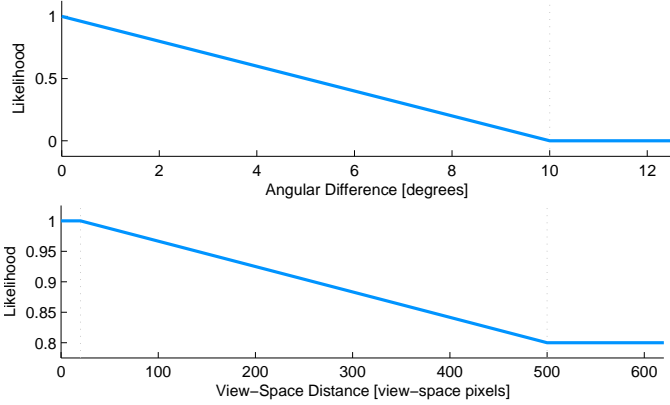


Figure A.22: Angular difference evaluation (top) and view-space distance evaluation (bottom) in *Line Parallelism* and *Line Perpendicularity* rules.

685 Appendix A.6. Line Length Equality

686 We measure the line length difference relative to a tested
687 line segment to get the likelihood \mathcal{L}_{dff} (Figure A.23 top) and
688 also the likelihood \mathcal{L}_{dst} (Figure A.23 bottom) based on rela-
689 tive distances of existing line segments to the tested one. Simi-
690 larly to line parallelism rule, the final likelihood is computed
691 as $\mathcal{L}_{LLE} = \mathcal{L}_{dff}\mathcal{L}_{dst}$ and an output suggestion is produced, if
692 $\mathcal{L}_{LLE} > 0.7$.

693 Appendix A.7. Arc and Circle Center Snapping

694 Based on the arc’s span θ and radius r , we first determine the
695 search distance $D' = \max(D, 2r(1 - \theta/2\pi))$, where the default
696 distance $D = 30$ (view-space pixels) is equal to the one used in
697 endpoint snapping and also the final likelihood \mathcal{L}_{ACCS} is then
698 computed using the same ramp function (Figure A.20).

699 Appendix A.8. Path Identity

700 We compute the discrete Fréchet distance between the tested
701 path and the existing one, as described in Section 3.4. The abso-
702 lute distance δ_F is then made relative to the length of the tested
703 path and used to compute the likelihood \mathcal{L}_{PI} (Figure A.24).

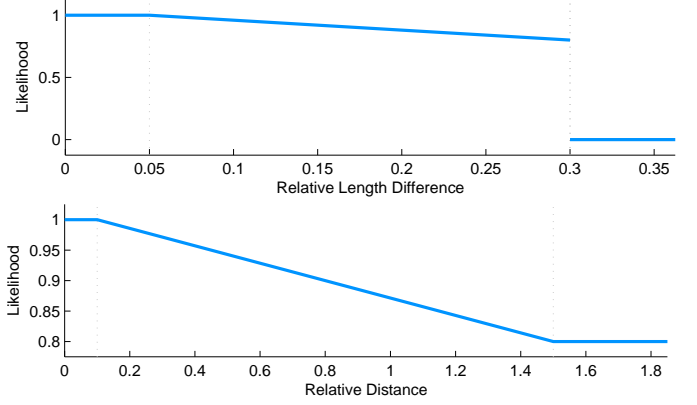


Figure A.23: Relative length difference evaluation (top) and relative distance evaluation (bottom) in *Line Length Equality* rule.

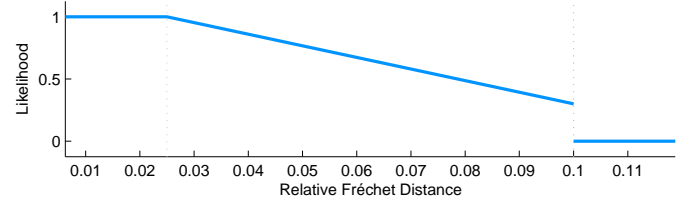


Figure A.24: Relative discrete Fréchet distance evaluation in *Path Identity* rule.

704 Appendix A.9. Path Transformation Adjustment

705 We compute four separate likelihoods for the angular dif-
706 ference \mathcal{L}_a , the scale difference \mathcal{L}_s , the x offset \mathcal{L}_x , and the y
707 offset \mathcal{L}_y , and perform only those with non-zero values. The
708 final likelihood is $\mathcal{L}_{TA} = 1 - (1 - \mathcal{L}_a)(1 - \mathcal{L}_s)(1 - \mathcal{L}_x)(1 - \mathcal{L}_y)$.
709 Note that the maximum likelihood is relatively small compared
710 to other rules; if they were larger, this would usually overwhelm
711 the likelihoods of other suggestions.

712 Appendix A.10. Path Offset

713 The process to obtain samples along the tested path together
714 with their signed distances to the existing path is described
715 in Section 3.6. To compute the likelihood \mathcal{L}_{PO} we evaluate
716 the relative distance difference between 25th and 75th quantile
717 from the sorted hit data (Figure A.26).

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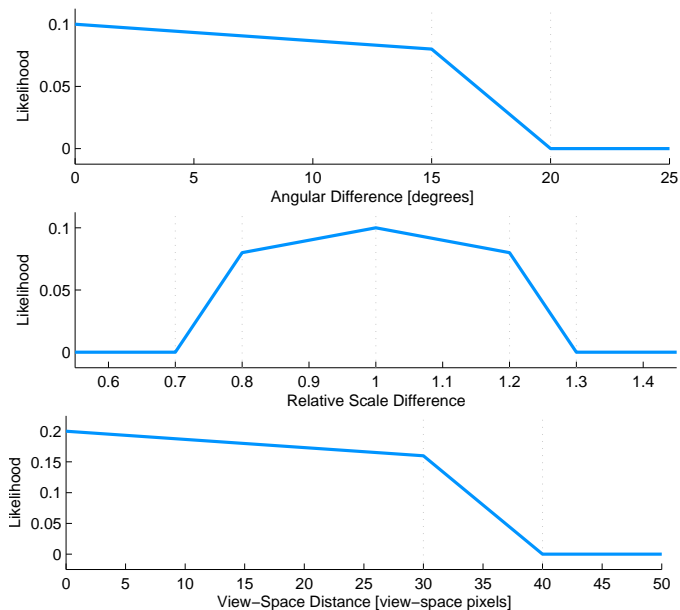


Figure A.25: Angular difference evaluation (top), relative scale evaluation (middle) and view-space distance evaluation (bottom) in *Transform Adjustment* rule.

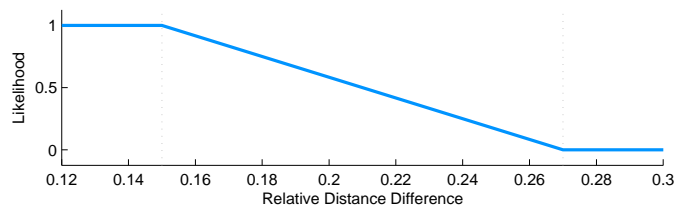


Figure A.26: Relative distance difference evaluation in *Path Offset* rule.

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